Solution

We must evaluate $\int_0^{+\infty} t\left(0.01e^{-0.01t}\right)dt$. We use either integration by parts or formula 20 from Table 1 in Section 7.3 to get the antiderivative

$$-te^{-0.01t}-\frac{1}{0.01}e^{-0.01t}=-\frac{t}{e^{0.01t}}-\frac{100}{e^{0.01t}}.$$

Now we evaluate the integral using limits: $\mu=\lim\limits_{b\to+\infty}\left(-\frac{b}{e^{0.01b}}-\frac{100}{e^{0.01b}}\right)-\left(0-100\right)=100$, since the limit expression goes to zero. Thus, the average duration of a hair appointment is 100 minutes, or 1 hour and 40 minutes.

Note that the constant k in the probability density function for an exponential distribution is all that is needed to find the mean value; that is, $\mu=\frac{1}{k}$.

7.5 EXERCISES

PRACTICE

In Exercises 1–12, show that each function is a probability density function on the given interval. If the function is not a probability density function on the given interval, explain why.

1. $f(x)=\frac{3}{16}\sqrt{x},\ [0, 4]$

2. $f(x)=\frac{4}{45}\sqrt[3]{x},\ [1, 8]$

3. $f(x)=\frac{4}{15}\left(\frac{1}{2}x+1\right),\ [-1, 2]$

4. $f(x)=\frac{1}{4}(x+1),\ [0, 2]$

5. $f(x)=\frac{3}{68}\left(x-\sqrt{x}\right),\ [1, 9]$

6. $f(x)=\frac{2}{13}\left(x+\frac{2}{x}\right),\ [2, 4]$

7. $f(x)=\frac{1}{2x},\ \left[1, e^2\right]$

8. $f(x)=\frac{1}{3x},\ \left[1, e^3\right]$

9. $f(x)=\frac{3}{32}\left(4-x^2\right),\ [-2, 2]$

10. $f(x)=6\left(\sqrt{x}-x\right),\ [0, 1]$

11. $f(x)=2e^{-2x},\ [0, +\infty)$

12. $f(x)=\frac{1}{10}e^{-0.1x},\ [0, +\infty)$

In Exercises 13–20, **a.** determine the value of *k* such that the function is a probability density function on the given interval and **b.** determine the average or expected value of *x*.

13. $f(x)=k(3-x),\ [0, 3]$

14. $f(x)=k(5-2x),\ [-1, 2]$

15. $f(x)=\frac{k}{\sqrt{x}},\ [1, 4]$

16. $f(x)=ke^{2x},\ [0, 1]$

17. $f(x)=ke^{-0.25x},\ [0, +\infty)$

18. $f(x)=ke^{-0.5x},\ [2, +\infty)$

19. $f(x)=\frac{k}{x^3},\ [1, 4]$

20. $f(x)=\frac{k}{(x+1)^2},\ [0, 7]$

21. An experiment has the probability density function $f(x)=\frac{2}{9}(3x-x^2)$, where $0 \le x \le 3$. Find the probability that x is between 1 and 2.

22. The probability density function for an experiment is $f(x)=\frac{1}{3x}$, where $1 \le x \le e^3$. Find the probability that x is between 1 and 10.

23. The exponential probability density function for an experiment is given by $f(x)=0.4e^{-0.4x}$, where $x \ge 0$. What is the probability that $x \ge 10$?

24. The outcomes of an experiment are distributed exponentially according to $f(x)=0.7e^{-0.7x}$, where $x \ge 0$. What is the probability that $10 \le x \le 20$?

APPLICATIONS

25. **Waiting time:** The average waiting time in minutes for a shuttle at the airport parking lot has the probability density function $f(t)=\frac{1}{20}$, where $0 \le t \le 20$. What is the probability that you will wait at least 12 minutes? What is the average waiting time?

26. **Travel time:** The length of time t (in minutes) it requires Frank to get to work ranges from 25 to 40 minutes. The probability density function is $f(t)=\frac{1}{15}$, where $25 \le t \le 40$. If Frank allots himself 36 minutes to get to work, what is the probability that he will be late? If he leaves his house at 8:00 a.m., what is his average arrival time?

27. **Reaction time:** The length of time t (in seconds) that it takes the body to react to the injection of a particular drug has the probability density function $f(t)=\frac{81}{40t^3}$, where $1 \le t \le 9$. What is the probability that it will take at least 6 seconds for the body to react to the drug? What is the average waiting time for the body to react?

28. **Waiting time:** The length of time t (in minutes) that you and your friends will wait to be seated at a popular restaurant has the probability density function $f(t)=\frac{26}{25(t+1)^2}$, where $0 \le t \le 25$. What is the probability that you will wait between 5 and 10 minutes?

29. **Reliability:** A particular TV model has a 2-year warranty. The probability density function for the number of years t that this TV model will last without needing any repairs is $f(t)=0.15e^{-0.15t}$. What is the probability that this type of TV will need repairs before it is 2 years old?

30. **Traffic spacing:** On the freeway, the distance x (in feet) between your car and the car behind you has the probability density function $f(x)=0.02e^{-0.02x}$, where $x \ge 0$. What is the probability that the car behind you is within 30 feet of you?