Example 4: Integration by Parts

Evaluate the definite integral $\int_1^5 x\sqrt{x-1}dx$.

Solution

$u = x$	$dv = \sqrt{x-1}dx$
$du = dx$	$v = \int (x-1)^{\frac{1}{2}} dx = \frac{2}{3}(x-1)^{\frac{3}{2}}$

Now we evaluate the definite integral using the formula for integration by parts.

$$\int udv = uv - \int vdu$$

$$\begin{aligned}
\int_1^5 x\sqrt{x-1}dx &= x\cdot\frac{2}{3}(x-1)^{\frac{3}{2}}\Big]_1^5 - \int_1^5 \frac{2}{3}(x-1)^{\frac{3}{2}}\,dx \\
&= \frac{2}{3}x(x-1)^{\frac{3}{2}}\Big]_1^5 - \frac{2}{3}\cdot\frac{2}{5}(x-1)^{\frac{5}{2}}\Big]_1^5 \\
&= \frac{2}{3}x(x-1)^{\frac{3}{2}} - \frac{4}{15}(x-1)^{\frac{5}{2}}\Big]_1^5 \\
&= \left[\frac{10}{3}(4)^{\frac{3}{2}} - \frac{4}{15}(4)^{\frac{5}{2}}\right] - (0) \\
&= \frac{80}{3} - \frac{128}{15} \\
&= \frac{400}{15} - \frac{128}{15} \\
&= \frac{272}{15}
\end{aligned}$$

7.1 EXERCISES

PRACTICE

In Exercises 1–16, use the technique of integration by parts to find the integrals.

1. $\int xe^{2x}dx$

2. $\int 3xe^{-x}dx$

3. $\int 2ye^{0.5y}dy$

4. $\int 5te^{0.4t}dt$

5. $\int \ln t\,dt$

6. $\int y^2 \ln y\,dy$

7. $\int x^3 \ln 5x\,dx$

8. $\int 8x\ln 3x\,dx$

9. $\int x\sqrt{x+2}\,dx$

10. $\int x\sqrt{x-3}\,dx$

11. $\int x(x+4)^{-2}dx$

12. $\int x(x-1)^{-3}dx$

13. $\int \frac{t}{2e^{0.6t}}dt$

14. $\int y^2e^{3y}dy$

15. $\int \sqrt{x}\ln 7x\,dx$

16. $\int 3x(x-6)^{-\frac{2}{3}}\,dx$

In Exercises 17–22, use the technique of integration by parts to evaluate each definite integral. Round your answer to the nearest hundredth.

17. $\int_0^2 xe^{-2x}\,dx$

18. $\int_0^3 (x+1)e^{-0.5x}\,dx$

19. $\int_0^1 (x+2)e^{-4x}\,dx$

20. $\int_0^4 (1-2x)e^{1.2x}\,dx$

21. $\int_{-2}^3 \frac{x}{\sqrt{6+x}}\,dx$

22. $\int_0^4 x\sqrt{1+2x}\,dx$

In each of Exercises 23–30, identify the *u* and *dv* which would solve the integral using integration by parts. Then evaluate the integral and round your answer to the nearest hundredth.

23. $\int_0^1 4x(3x+1)^5\,dx$

24. $\int_1^2 \frac{x}{\sqrt{2x+5}}\,dx$

25. $\int_{-1}^2 (x+1)(x+2)^{\frac{3}{2}}\,dx$

26. $\int_1^4 \sqrt{x}\ln x\,dx$

27. $\int_1^5 x^2\ln x\,dx$

28. $\int_1^3 \frac{\ln t}{t^2}\,dt$

29. $\int_0^6 \ln(x+1)\,dx$

30. $\int_1^2 (2x+1)\ln x\,dx$

In Exercises 31–40, use the technique of substitution or integration by parts to evaluate the integrals.

31. $\int 5te^{-2t}\,dt$

32. $\int 5te^{-2t^2}\,dt$

33. $\int \sqrt{3x}\ln x\,dx$

34. $\int \frac{\ln x}{x}\,dx$

35. $\int 3x(2x^2-1)^{\frac{3}{2}}\,dx$

36. $\int 3x(2x-1)^{\frac{3}{2}}\,dx$

37. $\int \frac{(\ln x)^2}{x}\,dx$

38. $\int x\ln x^2\,dx$

39. $\int \frac{e^x}{1-e^x}\,dx$

40. $\int \frac{x}{\sqrt{5x^2-3}}\,dx$

APPLICATIONS

41. Demand for a natural resource: The demand for a natural resource *t* years from now will be increasing at a rate of $te^{0.01t}$ million units per year. If the current demand is 80 million units, write a function for the demand *t* years from now.

42. Revenue: The marginal revenue for *x* units of a product is given by $R'(x) = (200-30x)e^{-0.15x}$ dollars per unit. Find the revenue function $R(x)$ if $R(0) = 0$.

43. Revenue: The marginal revenue for *x* units of a product is given by $R'(x) = 18 - 0.4\ln x$ dollars per unit, where $x \geq 1$. Find the revenue function if $R(1) = \$18.40$.

44. Resale value: The value of a machine depreciates at a rate of $-200t(t+1)^{-2}$ dollars per year, where *t* is the age (in years) of the machine. If the original cost of the machine is \$540, find a function for the value of the machine when it is *t* years old.