

Thus the final form for $P(t)$ is $P(t) = \frac{10,000}{1 + 99e^{-0.14t}}$.

Table 1 contains the general solutions of differential equations that arise in three basic growth applications.

Application	Differential Equation	General Solution
Unbounded growth	$\frac{dy}{dt} = ky$	$y = Ce^{kt}$
Bounded growth	$\frac{dy}{dt} = k(M - y)$	$y = M + Ce^{-kt}$
Logistic curve	$\frac{dy}{dt} = ky(M - y)$	$y = \frac{M}{1 + Ce^{-Mkt}}$

TABLE 1: General Solutions for Growth Applications

6.7 EXERCISES

PRACTICE

In Exercises 1–12, verify that the differential equation has the given function as a particular solution.

1. $\frac{dy}{dx} = 6$, $y = 6x - 1$

2. $\frac{dy}{dx} = 3x - 2$, $y = \frac{3}{2}x^2 - 2x - 4$

3. $\frac{dy}{dx} = 3 + y$, $y = e^x - 3$

4. $x \frac{dy}{dx} - x + 2 = 0$, $y = x - 2 \ln x + 7$

5. $\frac{dy}{dx} = y^{\frac{3}{2}}$, $y = \frac{4}{(5-x)^2}$

6. $\frac{dy}{dx} = -0.5y$, $y = 3e^{-0.5x}$

7. $2 \frac{dy}{dx} + 3y = 1$, $y = \frac{1}{3} - 2e^{-1.5x}$

8. $x \frac{dy}{dx} = xy + y$, $y = 4xe^x$

9. $2x^2y'' - xy' - 2y = 4 - 15x$, $y = 3x^2 + 5x - 2$

10. $x^2y'' + xy' - y + \ln x = 0$, $y = x + \ln x$

11. $x^2y'' - xy' + y = 0$, $y = x \ln x$

12. $y'' - 2y' + y = 0$, $y = e^x(x + 2)$

In Exercises 13–20, find the solution of each separable differential equation.

13. $\frac{dy}{dx} = 3x + \frac{1}{x}$

14. $\frac{dy}{dx} = -3xy$

15. $\frac{dy}{dx} = \frac{2y-1}{x+1}$

16. $x \frac{dy}{dx} = \frac{x^2 + 1}{y^2}$

17. $\frac{dy}{dx} = -0.4y$

18. $\frac{dy}{dx} = -2(26 - y)$

19. $\frac{dy}{dx} = (1 - 5x)y^2$

20. $\frac{dy}{dx} = (x^2 + 1)e^{-y}$

In Exercises 21–32, solve each initial-value problem or obtain a general solution as indicated. (Refer to Table 1 in the text if necessary.)

21. $\frac{dy}{dx} = 0.6y(20 - y)$

22. $\frac{dy}{dx} = 0.3y(50 - y)$

23. $\frac{dy}{dx} = 0.25y$, $y = 5$ when $x = 0$

24. $\frac{dy}{dx} = -3x$, $y = 10$ when $x = 2$

25. $x \frac{dy}{dx} = y + 1$, $y = 14$ when $x = 3$

26. $\frac{dy}{dx} = -2xy$, $y = 18$ when $x = 0$

27. $\frac{dy}{dx} = -6x^2y^2$, $y = 2$ when $x = 1$

28. $\frac{dy}{dx} = \frac{xy}{x^2 + 1}$, $y = 7$ when $x = 0$

29. $\frac{dy}{dx} = 8x + 2xy$, $y = 10$ when $x = 0$

30. $\frac{dy}{dx} = 0.3(80 - y)$, $y = 60$ when $x = 0$

31. $\frac{dy}{dx} = 0.8y(40 - y)$, $y = 30$ when $x = 0$

32. $\frac{dy}{dx} = 0.04y(60 - y)$, $y = 10$ when $x = 0$

APPLICATIONS

33. **Elasticity of demand:** The elasticity of demand for a product is given by $E = 1.5$. Find the demand function $p = D(x)$ if $D(8) = 24$.

34. **Elasticity of demand:** The elasticity of demand for a product is given by $E = 2$. Find the demand function $p = D(x)$ if $D(25) = 30$.

35. **Elasticity of demand:** The elasticity of demand for a product is given by $E = \frac{2(120 - x)}{x}$. Find the demand function $p = D(x)$ if $D(20) = 180$.

36. **Elasticity of demand:** The elasticity of demand for a product is given by $E = \frac{60 - 0.4x}{0.2x}$. Find the demand function $p = D(x)$ if $D(70) = 28$.

- 37. Resale value:** The resale or salvage value V of a machine decreases at a rate proportional to its value. Thus $\frac{dV}{dt} = -kV$, where t is the machine's age in years and k is its rate of decrease in value.
- Find the expression for the value when the machine is t years old if the original value was \$24,000 and the rate of decrease is 6 percent.
 - Find the value of the machine when it is 7 years old.
- 38. Drug concentration:** The amount A of a drug remaining in a body t hours after an injection decreases at a rate proportional to the amount present. This suggests the differential equation $\frac{dA}{dt} = -kA$. The amount of a certain drug decreases at a rate of 3 percent per hour. Find the amount of the drug remaining in the body 4 hours after an injection of 20 cc of the drug.
- 39. Newton's Law of Cooling:** Newton's Law of Cooling states that the rate at which the temperature T of an object changes is proportional to the difference between the temperature of the object and the temperature of the surrounding medium. That is $\frac{dT}{dt} = -k(T - M)$, where k is the constant of proportionality, t is time, and M is the constant temperature of the medium.
- Solve the differential equation for $T(t)$.
 - Find $T(5)$, if $T(0) = 78^\circ$, $M = 26^\circ$, and $k = 0.3$.
- 40. Newton's Law of Cooling:** The temperature of a roast was 160° when it was removed from an oven and placed in a room with constant temperature of 76° . After 10 minutes, the temperature of the roast was 152° . Find the temperature 20 minutes after the roast was removed from the oven. (See Exercise 39.)
- 41. Spread of a rumor:** In a small community with a population of 2800, a rumor about the mayor was started. The rate at which the rumor spread was approximated by $\frac{dN}{dt} = 0.0003N(2800 - N)$ people per day, where N is the number of people who have heard the rumor t days after the rumor was started.
- Write an equation for $N(t)$, assuming that 20 people have heard the rumor at $t = 0$.
 - How many days will it take for 1500 people to hear the rumor? Round to the nearest day.
- 42. Spread of a disease:** The population of seals on an island is about 600. Biologists estimate that there are 12 seals with a very infectious disease. The disease will spread at a rate $\frac{dN}{dt} = 0.006N(600 - N)$ seals per day, where N is the number of infected seals t days after the discovery of the disease.
- Write a function $N(t)$ for the number of seals infected t days after the discovery of the disease.
 - At what time t will 300 seals will be infected? Round to the nearest day.