1. The domain is [0, 1].

2. The range is [0, 1].

3. $f(0) = 0$ and $f(1) = 1$.

4. $f(x) \le x$ for all x in the interval [0, 1].

Example 4: Using a Lorenz Curve

Suppose that $f(x) = 0.6x^2 + 0.4x$ represents a Lorenz curve for some country.

a. What percent of the country's total income is earned by the lower 50 percent of the families in this country?

b. Find the coefficient of inequality.

Solution

a. $f(0.5) = 0.6(0.5)^2 + 0.4(0.5) = 0.35$

Since we want the lower 50% of families, set $x = 0.5$.

The lower 50 percent of the families earn 35 percent of the country's total income.

b. $2\int_0^1 \left[x - \left(0.6x^2 + 0.4x\right)\right] dx$

$= 2\int_0^1 \left[\left(0.6x - 0.6x^2\right)\right] dx$

$= 2\left(\frac{0.6x^2}{2} - \frac{0.6x^3}{3}\right)\Big]_0^1$

$= 2\left[\left(\frac{0.6(1)^2}{2} - \frac{0.6(1)^3}{3}\right) - \left(\frac{0.6(0)^2}{2} - \frac{0.6(0)^3}{3}\right)\right]$

$= 2\left[(0.3 - 0.2) - 0\right]$

$= 0.2$

Combine terms in the integrand before finding the antiderivative.

The coefficient of inequality is 0.2.

6.6 EXERCISES

PRACTICE

In Exercises 1–16, find the area of the region bounded by the graphs of the given equations.

1. $y = x^2,\ y = x - 1,\ x = -1,\ x = 4$

2. $y = x^2 + 2,\ y = x,\ x = 2,\ x = 5$

3. $y = x^3,\ y = x^2,\ x = 0,\ x = 1$

4. $y = x^2 + 1,\ y = 1 - 2x,\ x = 0,\ x = 3$

5. $y=\sqrt{2x+1},\ y=3x+2,\ x=0,\ x=2$

6. $y=e^{x-1},\ y=x,\ x=1,\ x=4$

7. $y=e^{-x},\ y=x+1,\ x=0,\ x=3$

8. $y=x^2+1,\ y=e^{-0.2x},\ x=0,\ x=4$

9. $y=\dfrac{1}{x},\ y=\dfrac{5}{2}-x,\ x=\dfrac{1}{2},\ x=2$

10. $y=\dfrac{1}{x+1},\ y=e^{0.7x},\ x=0,\ x=2$

11. $y=x+1,\ y=x^2+x$

12. $y=x^2+1,\ y=6-x$

13. $y=\sqrt{x},\ y=x^2$

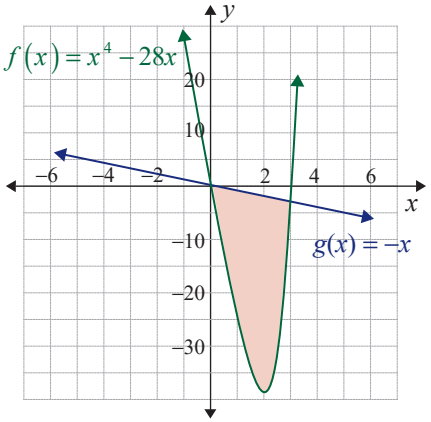
14. $y=x^2-6x,\ y=-x^2$

15. $y=x^2-2x-3,\ y=2x+2$

16. $y=x^2+5x-1,\ y=2-x^2$

For Exercises 17–21, determine the area pictured (check each answer using a graphing calculator if possibe). In each case, you must determine the limits of integration if necessary.

17.



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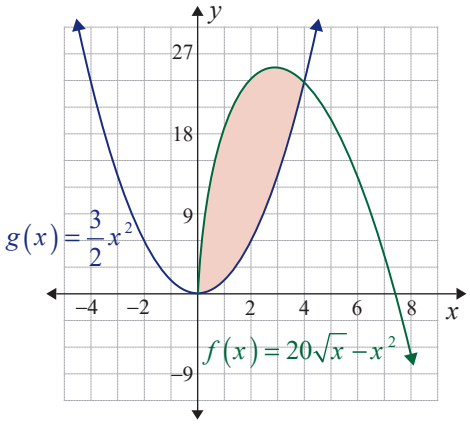

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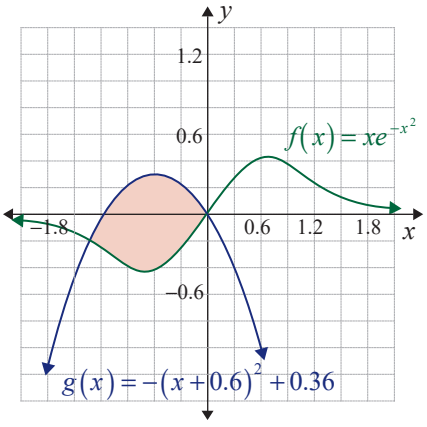

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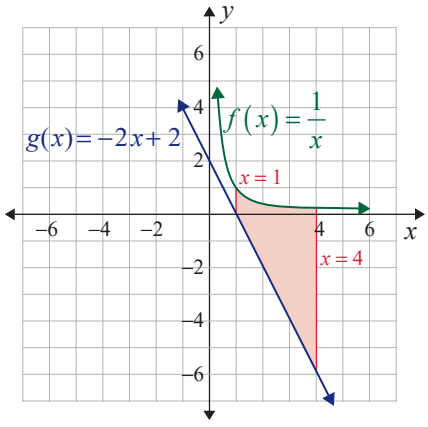

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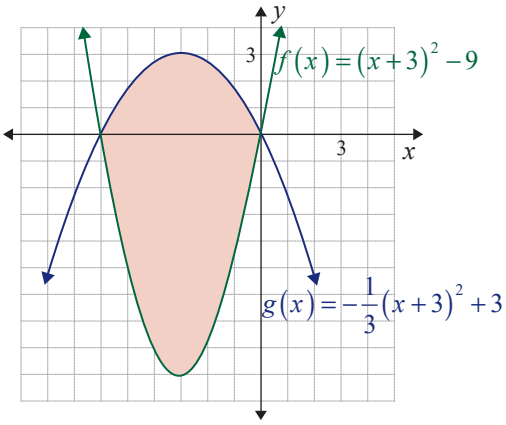


For each of the demand and supply functions in Exercises 22–25, find **a.** the equilibrium point, **b.** the consumers' surplus, and **c.** the producers' surplus.

22. $D(x)=18-0.4x,\ S(x)=3+0.1x,\ 0\le x\le 40$

23. $D(x)=24-0.2x,\ S(x)=10+0.5x,\ 0\le x\le 100$

24. $D(x)=1000-30x,\ S(x)=200+0.5x^2,\ 0\le x\le 30$

25. $D(x)=66-5\sqrt{x},\ S(x)=16+x,\ 0\le x\le 120$

APPLICATIONS

26. Consumers' surplus: The demand function for a particular product is given by the function $D(x)=24-0.6x-0.03x^2$. If $x_E = 10$ units, find the consumers' surplus.

27. Consumers' surplus: Find the consumers' surplus for a product if the demand function is given by $D(x)=\dfrac{800}{x+4}$ and $x_E = 4$ units.

28. Producers' surplus: Find the producers' surplus for a product if the supply function is given by $S(x)=9e^{0.4x}$ and $x_E = 5$ units.

29. Producers' surplus: The supply function for a product is given by the function $S(x)=\sqrt{16+1.5x}$. If $x_E = 6$ units, find the producers' surplus.

30. Consumers' and producers' surplus: The demand curve for a product is given by $D(x) = 18 - 3x$ and the corresponding supply curve is $S(x) = 3x + 6$. Find the consumers' surplus and the producers' surplus.

31. Consumers' and producers' surplus: The demand curve for a product is given by $D(x) = 125 - 15x$ and the corresponding supply curve is $S(x) = 50 + 10x$. Find the equilibrium point, the CS, and the PS.

32. Lorenz curve: The income distribution of a small country is estimated by the Lorenz curve $f(x) = \frac{13}{18}x^2 + \frac{5}{18}x$.

a. What percentage of the country's total income is earned by the lower 80 percent of its families? Round to the nearest percentage point.

b. Find the coefficient of inequality.

33. Lorenz curve: The Lorenz curve for estimating the income distribution of a country is given by $f(x) = \frac{7}{16}x^2 + \frac{9}{16}x$.

a. What percentage of the country's total income is earned by the lower 70 percent of its families? Round to the nearest percentage point.

b. Find the coefficient of inequality.

34. Lorenz curve: A study shows that the income distribution of farmers in a certain state is estimated by $f(x) = 0.47x^3 + 0.24x^2 + 0.29x$.

a. What percentage of the state's farming income is earned by the lower 60 percent of the state's farmers? Round to the nearest percentage point.

b. Find the coefficient of inequality.

35. Lorenz curve: In a certain state the income distribution for the lumber and the logging industry is estimated by $f(x) = 1.16x^3 - 0.82x^2 + 0.66x$.

a. What percentage of the state's lumber and logging income is earned by the lower 50 percent of the companies? Round to the nearest percentage point.

b. Find the coefficient of inequality.

36. Lorenz curve: The income distribution for a certain country in 1996 was estimated by the function $f(x) = 0.34x + 0.66x^2$. In 2000, the income distribution was estimated by the function $f(x) = 0.3x + 0.72x^2 - 0.02x^3$.

a. Find the coefficient of inequality for each of the years.

b. Which year had a more equitable income distribution?

37. Lorenz curve: The income distribution for country A is estimated by the function $f(x) = 0.24x + 0.72x^2 + 0.04x^3$. The income distribution for country B is estimated by the function $f(x) = 0.28x + 0.69x^2 + 0.03x^3$.

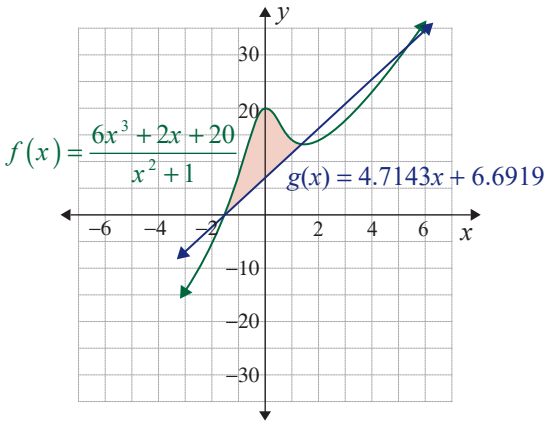
a. Find the coefficient of inequality for each of the two countries.

b. Which country has a more equitable income distribution?

TECHNOLOGY

For Exercises 38–41, use a graphing calculator to determine the area.

38.



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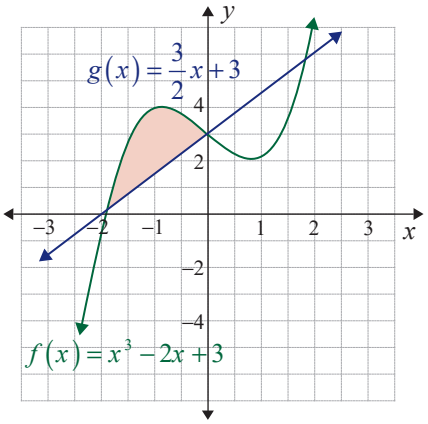

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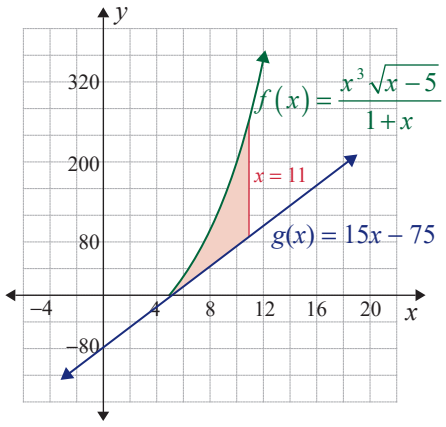

41.