

b. Integrate from $x = 200$ to $x = 400$.

$$\begin{aligned} \int_{200}^{400} (20 - 0.05x) dx &= 20x - 0.025x^2 \Big|_{200}^{400} \\ &= [20(400) - 0.025(400)^2] - [20(200) - 0.025(200)^2] \\ &= (8000 - 4000) - (4000 - 1000) \\ &= 1000 \end{aligned}$$

His profit changes by \$1000 when sales increase from 200 to 400 frames.

Note that from parts a. and b. we see that the profit on sales for the first 200 frames ($x = 0$ to $x = 200$) is greater than the profit for the second 200 frames ($x = 200$ to $x = 400$). This result is quite reasonable because the marginal profit, $P'(x) = 20 - 0.05x$, is decreasing by 5 cents per frame. In this problem, the fixed costs are relevant. Suppose the fixed costs are \$1000. Then $P(x) = 20x - 0.025x^2 - 1000$ dollars, and $P(200) = \$2000$, which is the **net profit**. The integral from $x = 0$ to $x = 200$ gives the increase in profits, $P(200) - P(0) = 2000 - (-1000) = 3000$.

6.5 EXERCISES

PRACTICE

For Exercises 1–18, find the total area bounded by the x -axis and the curve $y = f(x)$ on the indicated interval.

- $f(x) = 3x + 1$, $[0, 5]$
- $f(x) = 7 - 2x$, $[-1, 3]$
- $f(x) = x^2 + 1$, $[-2, 2]$
- $f(x) = 0.5x^2 + 2$, $[1, 4]$
- $f(x) = x^3 + 2$, $[-1, 1]$
- $f(x) = 2x^3 - 1$, $[1, 2]$
- $f(x) = x^2 + x + 1$, $[-1, 3]$
- $f(x) = x^2 + 2x - 3$, $[1, 3]$
- $f(x) = \frac{4}{x+1}$, $[0, 3]$
- $f(x) = \frac{3}{2x+1}$, $[0, 2]$
- $f(x) = 3e^{0.6x}$, $[0, 5]$
- $f(x) = 1 + e^{-0.3x}$, $[0, 4]$
- $f(x) = x^2 - 2x - 8$, $[2, 5]$
- $f(x) = x^2 + 3x - 4$, $[0, 4]$
- $f(x) = \begin{cases} 2 - x & \text{if } -1 \leq x \leq 2 \\ x^2 - 4 & \text{if } 2 \leq x \leq 3 \end{cases}$
- $f(x) = \begin{cases} x^2 & \text{if } -2 \leq x \leq 1 \\ 2x - 1 & \text{if } 1 \leq x \leq 2 \end{cases}$
- $f(x) = \begin{cases} x + 2 & \text{if } -2 \leq x \leq 0 \\ \sqrt{x+4} & \text{if } 0 \leq x \leq 5 \end{cases}$
- $f(x) = \begin{cases} 1 - 2x & \text{if } -2 \leq x \leq 0 \\ e^{2x} & \text{if } 0 \leq x \leq 1.5 \end{cases}$

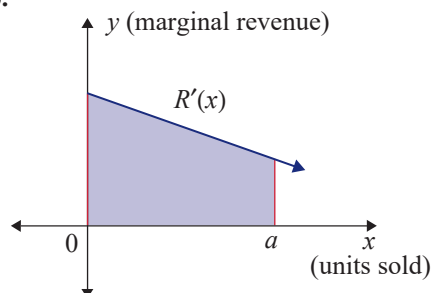
🔑 APPLICATIONS

- 19. Profit:** The marginal profit for a certain style of sports jacket is given by $P'(x) = 56 - 0.8x$ dollars per jacket, where x is the number of jackets produced and sold weekly. Find the profit for the first 50 jackets that are produced and sold. (Ignore any fixed costs.)
- 20. Profit:** The marginal profit of an important product is given by $P'(x) = 10 - 0.015e^{0.6x}$ dollars per item, where x is the number of items produced and sold. Find the profit for the first 8 items. (Ignore any fixed costs.)
- 21. Cost:** The marginal cost of a product is given by $15 + \frac{4}{\sqrt{x}}$ dollars per unit, where x is the number of units produced. The current level of production is 100 units weekly. If the level of production is increased to 169 units weekly, find the increase in the total costs.
- 22. Revenue:** The marginal revenue from the sale of x bottles of a wine is given by $8.4 - 0.3\sqrt{x}$ dollars per bottle. Find the increase in total revenue if the number of bottles sold is increased from 225 to 350.
- 23. Wildlife management:** The manager of a wildlife preserve has started a management program to control the population of the preserve's bison herd. It is estimated that the population will continue to grow according to the function $N'(t) = 15 - 6t^{\frac{1}{2}}$ bison per year, where t is the number of years after implementation of the plan and $0 \leq t \leq 5$. Find the increase in the population during the first 4 years of the program.
- 24. Bacterial population:** It is estimated that t hours after some particular bacteria are introduced into a culture, the population will be increasing at a rate of $P'(t) = \frac{1200}{(12 - 0.5t)^{\frac{1}{2}}}$ bacteria per hour. Find the increase in the population during the first 6 hours.

✎ WRITING & THINKING

In Exercises 25 and 26, explain the meaning of the shaded region in each graph.

25.



26.

