6.2 EXERCISES

PRACTICE

In Exercises 1–36, use the technique of substitution to perform each integration.

1. $\int (x+4)^7\,dx$

2. $\int (y-6)^{-3}\,dy$

3. $\int 2x\left(x^2-1\right)^{\frac{1}{2}}dx$

4. $\int 3x^2\left(x^3-5\right)^{\frac{1}{3}}dx$

5. $\int \frac{1}{t+2}\,dt$

6. $\int \frac{1}{x-11}\,dx$

7. $\int \frac{3t^2}{t^3+4}\,dt$

8. $\int \frac{1}{y^2-8}\cdot 2y\,dy$

9. $\int e^{y+5}\,dy$

10. $\int e^{x-9}\,dx$

11. $\int e^{-0.2x}(-0.2)\,dx$

12. $\int e^{0.5t}(0.5)\,dt$

13. $\int \frac{1}{5x+3}\,dx$

14. $\int (3y-2)^{-2}\,dy$

15. $\int \frac{1}{\sqrt{4x-1}}\,dx$

16. $\int e^{-4x}\,dx$

17. $\int xe^{2x^2}\,dx$

18. $\int \frac{x}{2x^2+5}\,dx$

19. $\int \frac{x}{\left(3x^2-1\right)^2}\,dx$

20. $\int y^2e^{-2y^3}\,dy$

21. $\int \frac{2t+1}{t^2+t-4}\,dt$

22. $\int \left(x^2+3x-1\right)^4(2x+3)\,dx$

23. $\int 5ye^{-y^2}\,dy$

24. $\int \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt$

25. $\int 6x\sqrt{5+2x^2}\,dx$

26. $\int \frac{e^t}{e^t-1}\,dt$

27. $\int \frac{4}{x^2}e^{\frac{1}{x}}\,dx$

28. $\int 4y\sqrt[3]{3y^2+7}\,dy$

29. $\int e^{2x}\left(1-3e^{2x}\right)^2\,dx$

30. $\int \frac{4x+10}{x^2+5x+2}\,dx$

31. $\int \frac{\ln x}{x}\,dx$

32. $\int \frac{\ln 4x}{x}\,dx$

33. $\int \frac{1}{x\ln x}\,dx$

34. $\int \frac{(\ln x)^2}{x}\,dx$

35. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

36. $\int \frac{7x}{e^{x^2}}\,dx$

In Exercises 37–40, divide first and then integrate.

37. a. $\int \frac{x-1}{x-2}\,dx$ $\left(\textbf{Hint: } \frac{x-1}{x-2} = 1 + \frac{1}{x-2}\right)$

b. $\int \frac{x+3}{x+1}\,dx$ $\left(\textbf{Hint: } \frac{x+3}{x+1} = 1 + \frac{2}{x+1}\right)$

38. a. Rework Exercise 37a and substitute $u = x - 2$.

b. Rework Exercise 37b and substitute $u = x + 1$.

39. $\int \frac{x+2}{x+4}\,dx$

40. $\int \frac{x+5}{x-3}\,dx$

In Exercises 41–44, find a function $f(x)$ given $f'(x)$ and one (x, y)-value.

41. $f'(x) = 10xe^{5x^2}$; $(0, 11)$

42. $f'(x) = \frac{3}{x-5}$; $(6, 5)$

43. $f'(x) = (2x+2)^5$; $(-1, 10)$

44. $f'(x) = \frac{12x^2}{(x^3+6)^2}$; $\left(0, -\frac{2}{3}\right)$

APPLICATIONS

45. Appreciation: The value V of a painting is increasing at a rate of $4500(25-1.8t)^{-\frac{3}{2}}$ dollars per year. The painting originally sold for \$1000.

a. Write a function for its value t years after the original sale.

b. What will the painting be worth 5 years after the original sale?

46. Skills development: It is estimated that after t weeks of practice, students in a typing class can increase their speed $24e^{-0.2t}$ words per minute for each additional week of practice.

a. If their initial speed was 0 words per minute ($S(0) = 0$), write a function to represent their speed after t weeks of practice.

b. How fast can they type after 10 weeks (to the nearest word)?

47. Price: After the NBA championship basketball game, the price of a souvenir cap changes at the rate of $-\frac{1}{2x}$ dollars per cap, where x is the number of caps sold (in thousands). Write a function for the price p if $p(10) = 12.85$.

48. Profit: The marginal profit from the production and sale of x units of a product is estimated to be $\frac{100}{1+0.5x} - 4.5$ dollars per unit. If $P(0) = -60$, find the profit function $P(x)$.

49. Daily production: Records show that t hours after starting work on a typical day, an employee can assemble bikes at a rate of $\frac{6t+15}{\sqrt{t^2+5t}}$ per hour. Find the daily production function $N(t)$ if $N(0)=0$.

50. Position of a particle: A particle is moving in a straight line with the velocity $v(t)=\sqrt{2t+7}$ feet per second, where t represents time in seconds. Find the position function $s(t)$ if $s(0)=0$.

51. Position of a projectile: The velocity of a projectile moving in a straight line t seconds after it is fired is given by $v(t)=36+\frac{60}{(t+1)^2}$ feet per second. Find the position function $s(t)$ if $s(1)=10$.

52. The marginal value for a tract of land is $V'=\frac{20}{2t+1}$, where t is time in years since 2000 and V is in thousands of dollars.

a. Determine the function V in terms of t if $V(0)=20$.
b. What was the value of the land in 2015?

53. A point mass moving on a horizontal axis has a deceleration given by $a(t)=-48(2t+1)^2$, where t is time in seconds and a is in feet per second per second.

a. If $v(0)=6$ ft/s, determine a velocity function $v(t)$.
b. If $s(0)=-\frac{3}{4}$ feet, determine a distance function $s(t)$.

54. The number of viewers of a new TV show grew at a rate $V'(t)=900e^{0.3t-4}$ all summer long, where V is the total number of viewers in thousands and t is the number of weeks since June 1st.

a. Determine $V(t)$ if $V(0)=100$.
b. At what rate is V changing when $t=5$?
c. When will the show hit 1,000,000 viewers ($V=1000$)?

55. Suppose in a medieval country, from 1200 to 1300, the life expectancy of a female serf changed at the rate $f'(t)=\frac{0.3}{1+0.01t}$, where t is time in years after 1200 and $f(t)$ is the average age of death.

a. What are the units of $f'(t)$?
b. Determine $f(t)$ if $f(0)=30$.
c. What was the life expectancy of a female serf in 1300?

56. A new evening school is growing at the rate of $p'(t)=\frac{150}{\sqrt{1+0.2t}}$, where $p(t)$ is the total evening school student population and t is the time in years after 2000. The initial enrollment was 1500 students.

a. How fast was enrollment changing in 2002?
b. What was the expected enrollment in 2005?