speed at $t = 0$. Thus $C_1 = -200$ m/s. We use -200 rather than $+200$ since the direction of the meteor is toward the origin (so s is decreasing due to the speed). Therefore,

$$v(t) = -30t^2 + 64t - 200.$$

Since $v = \frac{ds}{dt}$, we can say $ds = v \cdot dt$. Integrating again, we have

$$\int ds = \int v \cdot dt = \int \left(-30t^2 + 64t - 200\right) \cdot dt.$$

Integrating a second time gives

$$s = -30\left(\frac{1}{3}t^3\right) + 64\left(\frac{1}{2}t^2\right) - 200t + C_2$$
$$s = -10t^3 + 32t^2 - 200t + C_2.$$

Once again there is a constant of integration to evaluate. If at the time of initial measurement the distance of the meteor from Earth is 50,000 kilometers, then $s(0) = 50{,}000{,}000$ meters.

$$50{,}000{,}000 = s(0) = -10(0)^3 + 32(0)^2 - 200(0) + C_2 = C_2$$

Here we see $C_2 = 50{,}000{,}000$ meters, the initial distance of the meteor from Earth.

It is typical of "acceleration" problems that there are two integrations, two constants of integration to evaluate, and two additional pieces of data necessary for this evaluation. It is common to use the notation v_0 and s_0 to denote initial ($t = 0$) values of velocity and distance.

One case of special interest is a body falling due to Earth's gravity. In this case, the acceleration a is a constant. As before, $a = \frac{dv}{dt}$ so $dv = a \cdot dt$. Integrating both sides gives

$$v = at + C.$$

The constant C is v_0, the initial velocity. That is, $v = at + v_0$. Since $\frac{ds}{dt} = v$, $ds = v \cdot dt$. One more integration gives

$$s = \int ds = \int \left(at + v_0\right) dt = a\left(\frac{1}{2}t^2\right) + v_0 t + s_0$$
$$s = \frac{1}{2}at^2 + v_0 t + s_0.$$

6.1 EXERCISES

PRACTICE

In Exercises 1–12, show that the function $F(x)$ is an antiderivative of the function $f(x)$ by differentiating F.

1. $F(x) = 4x - 1, \quad f(x) = 4$

2. $F(x) = 6x, \quad f(x) = 6$

3. $F(x) = 3x^2 + 5x + 2, \quad f(x) = 6x + 5$

4. $F(x)=\frac{1}{2}x^2-4x+e^{2x}-1,\quad f(x)=x-4+2e^{2x}$

5. $F(x)=\ln x-\frac{1}{x}-4e^{x^2},\quad f(x)=\frac{1}{x}+\frac{1}{x^2}-8xe^{x^2}$

6. $F(x)=\ln x^3+\frac{1}{x^2}+6,\quad f(x)=\frac{3}{x}-\frac{2}{x^3}$

7. $F(x)=\left(x^2+3\right)^4-1,\quad f(x)=8x\left(x^2+3\right)^3$

8. $F(x)=3(5x-1)^{\frac{2}{3}}+8,\quad f(x)=\frac{10}{\sqrt[3]{5x-1}}$

9. $F(x)=\frac{5}{3}\left(e^x-4\right)^3+e,\quad f(x)=5e^x\left(e^x-4\right)^2$

10. $F(x)=3e^{x^2-1}-7,\quad f(x)=6xe^{x^2-1}$

11. $F(x)=\ln\left(x^2+5x-3\right)-\sqrt{5},\quad f(x)=\frac{2x+5}{x^2+5x-3}$

12. $F(x)=\ln\left(e^{3x}-x\right)+\sqrt{11},\quad f(x)=\frac{3e^{3x}-1}{e^{3x}-x}$

Find the indefinite integrals in Exercises 13–32.

13. $\int 7\,dx$

14. $\int \frac{2}{3}\,dx$

15. $\int 5x^4\,dx$

16. $\int -2x^{-3}\,dx$

17. $\int \left(x^2-3\right)dx$

18. $\int \left(x^4+5\right)dx$

19. $\int \left(\frac{1}{3}-e^t\right)dt$

20. $\int \left(e^t+t\right)dt$

21. $\int \left(\frac{1}{y}+y^3\right)dy$

22. $\int \left(\frac{1}{\sqrt{y}}-\frac{1}{y}\right)dy$

23. $\int \left(4x^2+\frac{2}{x}+\frac{1}{x^2}\right)dx$

24. $\int \left(9x-\frac{3}{x}-\frac{1}{\sqrt{x}}\right)dx$

25. $\int \left(2\sqrt[3]{x}+5\sqrt{x}\right)dx$

26. $\int \left(\sqrt{x}+6e^x-\frac{5}{x}\right)dx$

27. $\int \left(4e^y-2y^5-\frac{1}{5}\right)dy$

28. $\int \left(y^{\frac{3}{2}}+5y^{-\frac{2}{3}}-y^{-1}\right)dy$

29. $\int \left(\frac{2}{\sqrt[3]{t}}+\frac{7}{t^3}\right)dt$

30. $\int \left(2t^{\frac{5}{2}}+\frac{4}{t}-\sqrt{3}\right)dt$

31. $\int \left(\frac{2}{3}e^x+x^{-\frac{3}{2}}-7x^{-1}\right)dx$

32. $\int \left(0.25e^x+4x^{-\frac{1}{4}}\right)dx$

In Exercises 33–38, perform the indicated multiplication and then integrate.

33. $\int x^3(2x-1)\,dx$

34. $\int x^2(3x-5)\,dx$

35. $\int (3t+2)^2\,dt$

36. $\int (5x+6)^2\,dx$

37. $\int \sqrt{y}\left(y^2+2y-1\right)dy$

38. $\int \sqrt{y}\left(4-3y-2y^2\right)dy$

In Exercises 39–44, simplify the indicated quotient and then integrate.

39. $\int \frac{3x^2+5x-4}{x^2}\,dx$

40. $\int \frac{x^3-6x^2+x}{x^2}\,dx$

41. $\int \frac{4+\sqrt{x}-3x}{x}\,dx$

42. $\int \frac{5x^2-2x+3}{\sqrt{x}}\,dx$

43. $\int \frac{x^{\frac{3}{2}}+6-2xe^x}{x}\,dx$

44. $\int \frac{4x^2+4\sqrt{x}-7x}{x^2}\,dx$

45. Find the antiderivative $F(x)$ that satisfies the given condition.

a. $F'(x)=x^2-e^x, \quad F(0)=1$

b. $F'(x)=6x^2+x-10, \quad F(0)=0$

c. $\frac{dF}{dx}=\frac{10}{\sqrt{x}}, \quad F(1)=20$

d. $\frac{dF}{dx}=6e^x-2, \quad F(0)=-10$

46. Given that $f'(x)=x^2-2$, determine the function $f(x)$ with the given constant of integration C. Draw all three functions on the same coordinate system.
a. $C=-1$
b. $C=1$
c. $C=3$

47. Given $f'(x)=6x^2-24x$.
a. Determine the x-values at which $f(x)$ has a local maximum or minimum.
b. Determine whether there is an inflection point.
c. Given $f(1)=-9$, sketch f and determine if the answers to part a. are correct.

APPLICATIONS

48. Cost: The weekly marginal cost of producing x ice cream makers is $28 + 0.05x$ dollars per ice cream maker. Find the cost function if the fixed costs are \$2400.

49. Cost: The marginal cost of producing x clock radios is $0.3x^2-0.8x+24$ dollars per clock radio. The fixed costs are \$1500. Find the cost function.

50. Revenue: The marginal revenue from selling x irons is $94 - 0.06x$ dollars per iron. Find the revenue function. (**Hint:** $R(0) = 0$.)

51. Revenue: The marginal revenue from selling x floor lamps is $100 - 0.2x$ dollars per lamp. Find the revenue function. (**Hint:** $R(0) = 0$.)

52. Profit: The marginal profit from the production and sale of x cameras is estimated to be $24 - 0.4x$ dollars per unit.

a. Find the firm's profit function if the profit from the production and sale of 80 units is \$240.

b. What is the profit from the sale of 90 units?

53. Profit: A manufacturer has determined that the marginal profit from the production and sale of x wireless speakers is approximately $120 - 3x$ dollars per speaker.

a. Find the profit function if the profit from the production and sale of 30 speakers is \$1200.

b. What is the profit from the sale of 40 speakers?

54. Population: The population of a community is growing at a rate given by $\frac{dP}{dt} = 120 - 15t^{\frac{1}{2}}$ people per year. Find a function to describe the population t years from now if the present population is 8600 people.

55. Rodent control: Animal control officers have implemented a program to eliminate rats in a community. They estimate that the population of rats is changing at a rate of $\frac{dP}{dt} = 24t^{\frac{1}{2}} - 40t$ rats per month. Find a function for the rat population t months from now if the current population is estimated to be 6300 rats.

56. Air quality: The air quality control office estimates that for a population of x thousand people, the level of pollution in the air is increasing at a rate of $\frac{dL}{dx} = 0.2 + 0.002x$ parts per million per thousand people. Find a function to estimate the level of the pollutants if the level is 5.4 parts per million when the population is 20,000 people.

57. Ecology: Biologists are treating a stream contaminated with bacteria. The level of contamination is changing at a rate of $\frac{dN}{dt} = -\frac{960}{t^2} - 240$ bacteria per cubic centimeter per day, where t is the number of days since the treatment began. Find a function $N(t)$ to estimate the level of contamination if the level after 1 day was about 5720 bacteria per cubic centimeter.

58. Height: An object is projected vertically so that the velocity after t seconds is given by $v(t) = 96 - 32t$ feet per second.

a. Find the height function $s(t)$ if $s(0) = 18$ feet.

b. What will be the height after 3 seconds?

59. Distance: A vehicle travels in a straight line for t minutes with a velocity $v(t) = 72t - 6t^2$ feet per minute, for $0 \le t \le 10$.

a. Find the distance function $s(t)$ if $s(0) = 0$.

b. How far will the vehicle travel in 5 minutes?

c. How far will the vehicle travel in 10 minutes?

60. Distance: A particle moves along an axis with velocity given by $v(t) = 3t - 1$, where t is in seconds and v is in ft/s.

a. Determine the acceleration, $a(t)$.

b. What is the distance function, $s(t)$, if $s(0) = 5$?

61. Distance: A meteor falls partly under the influence of Earth's gravity at a velocity given by $v(t) = 200 + 30t + 24t^{\frac{1}{2}}$ for $0 \leq t \leq 24$, where t is in hours and v is in miles per hour.

a. Determine the acceleration.

b. Determine the distance function if $s(0) = 5000$ miles.

WRITING & THINKING

62. a. Compute the derivative of $y = e^{mx+b}$ where m and b are constants. Use your answer to determine an integration formula for $\int e^{mx+b}\,dx$.

b. Compute the derivative of $y = \frac{1}{e^{mx+b}}$ where m and b are constants. Use your answer to determine an integration formula for $\int \frac{1}{e^{mx+b}}\,dx$.