Thus, initially, there are 200 grams of the isotope present. We want to find the time it takes for there to be 100 grams left (half of the initial amount of 200 grams).

Therefore, we want to find $t$ in the equation

$$100 = 200e^{-0.05t}.$$

We divide both sides of the equation by 200 and then use the definition of natural logarithm:

$$\frac{100}{200} = \frac{200e^{-0.05t}}{200}$$

$$\frac{1}{2} = e^{-0.05t}$$

$$-0.05t = \ln\frac{1}{2}.$$

Thus,

$$t = \frac{\ln\frac{1}{2}}{-0.05} \approx 13.86.$$

The half-life of the isotope is about 13.86 years.

## 5.5 EXERCISES

### APPLICATIONS

1. **Population:** The population of a city is growing exponentially at a rate of 3.5 percent per year. The population was 8400 in 2000.
   - **a.** Find an exponential function that represents the population $t$ years after 2000.
   - **b.** What was the population in the year 2010?
   - **c.** When was the population 12,800?

2. **Bee population:** A swarm of bees grows exponentially at a rate of 4 percent hourly. Initially, there were 900 bees in the swarm.
   - **a.** Find an exponential function for the number of bees in the swarm after $t$ hours.
   - **b.** How many bees are in the swarm after 6 hours?
   - **c.** How many hours will it take for the swarm to double in size? Round your answer to the nearest tenth.

3. **Cost:** In 2018, the cost of a medium pizza was about $9.00. In 2021, the cost was $12.00. If the cost is growing exponentially, predict the cost of a medium pizza in 2027?

4. **Ant colony:** A colony of ants is growing exponentially. When first observed, the colony contained about 400 ants. If at the end of 9 days there are about 700 ants, approximately how many ants will be present at the end of 15 days?

5. **Cell growth:** A cell culture grows from 1000 to 1600 cells in 12 hours. If the rate of growth is proportional to the size, when will the culture contain 2000 cells?

6. **Bacterial population:** A bacteria culture grows at a rate proportional to its size. If the population doubles every 6 hours, how long will it take for the population to be three times its initial size?

7. **Demand for oil:** The demand for oil in the United States doubles every 8 years. How long will it take for the demand to triple?

8. **Inflation:** The amount of goods and services that costs \$100 on January 1, 2015 costs \$139.10 on January 1, 2018. Estimate the cost of the same goods and services on January 1, 2025. Assume the cost is growing exponentially.

9. **Interest compounded continuously:** One thousand dollars is deposited in a savings account where the interest is compounded continuously. After 4 years, the balance will be \$1366.15. When will the balance be \$1870.00?

10. **Half-life:** The decay rate for a radioactive isotope is 2.6 percent per year. Find its half-life.

11. **Half-life:** The decay rate of a radioactive isotope is 6.5 percent per year. Find its half-life.

12. **Half-life:** The half-life of a radioactive material is 1480 years. After how many years will only 20 percent of the material remain?

13. **Archaeological dating:** A wooden carving found at an archaeological dig contains about 34 percent of its carbon-14. Approximately how old is the carving?

14. **Archaeological dating:** Bones from the skeleton of an animal have lost 62 percent of their carbon-14. Estimate the age of the bones.

15. **Atmospheric pressure:** As the elevation above sea level is increased, the atmospheric pressure declines exponentially. The pressure at sea level is approximately 15 lb/in.$^2$ and the pressure at 3000 feet of elevation is about 13 lb/in.$^2$ Find the pressure at 5000 ft.

16. **Drug concentration:** The concentration of a drug in the body fluids is known to decline exponentially. If 20 mg of a drug is administered and 8 mg remains after 3 hours, how much will remain after 5 hours?

17. **Depreciation:** It is determined that the value of a piece of machinery declines exponentially. A machine that was purchased 5 years ago for \$65,000 is worth \$35,000 today. What will be the value of the machine 5 years from now?

18. **Population:** The population of a certain economically depressed union is declining exponentially at a rate of 1.5 percent. If the population in 2010 was 30,000, estimate the population in 2030.

19. **Reliability:** Studies show that the fraction $P$ of light bulbs that have burned out after $t$ hours of use is given by $P = 1 - e^{-0.03t}$. What fraction of the bulbs have burned out after 50 hours? How long will it be before half of the bulbs have burned out?

**20. Advertising:** A radio station estimates that during an intense advertising campaign, the number of people $N$ who will hear a commercial is given by $N = A\left(1 - e^{-0.02x}\right)$, where $A$ is the number of people in the broadcasting area and $x$ is the number of times the commercial is run. There are 60,000 people in the area.

**a.** How many people will hear the commercial if it is run 20 times?

**b.** How many times should the station plan to run the commercial to be certain that at least 30,000 people hear it?

**21. Ecology:** The Department of Fisheries has begun a reclamation project at a lake where the fish population was nearly destroyed by agricultural chemicals. They estimate that the population of fish in $t$ years will be $P = 6000 - 5200e^{-0.28t}$.

**a.** What was the initial population?

**b.** What will be the population after 4 years?

**c.** How long will it take for the population to be 5000 fish?

**22. Advertising:** The manager of The Sound Lab has determined that after an intense advertising campaign, the monthly sales of a particular wireless speaker can be approximated by $N = 300 + 180e^{-0.04t}$ units, where $t$ is the number of months after the campaign.

**a.** Find the monthly sales initially.

**b.** Find the monthly sales when $t = 6$.

**c.** When will the monthly sales be 400 units?

**23. Skills development:** Beverly is making a small souvenir to give to each person attending her family reunion. The length of time, in minutes, she takes to make the $n^{\text{th}}$ one is given by the function $T(n) = 12 + 30e^{-0.1n}$. How long will it take her to make the $30^{\text{th}}$ souvenir?

**24. Dairy farming:** The number of dairy farmers in a particular state who are feeding a new supplement to their milking cows is given by the function $W(t) = 340\left(1 - e^{-0.09t}\right)$, where $t$ is the number of months the supplement has been available. How long will it be before 200 farmers are feeding the supplement to their cows?

**25. Cost:** The total cost function for a local company is given by $C(t) = 12 - ce^{-kt}$ in thousands of dollars, where $t$ is the time in months. The fixed costs are \$5000 and the total cost after 2 months is \$10,200. Find the total cost at the end of 6 months.

**26. Skills development:** The time that it takes a service attendant to change a tire is given by the function $T(x) = 4.4 + Ce^{-kx}$ minutes, where $x$ is the number of tires the attendant has changed before. It takes Patrick 15 minutes to change the first tire ($x = 0$) and 9.3 minutes to change the seventh tire. How long will it take him to change the eleventh tire?