$$\begin{aligned}\frac{dy}{dx} &= (\ln 10)\cdot\left(10^{-2x}\right)\cdot\frac{d}{dx}(-2x)\\ &= (\ln 10)\cdot\left(10^{-2x}\right)\cdot(-2)\\ &= -2\ln 10\cdot\left(10^{-2x}\right)\end{aligned}$$

5.4 EXERCISES

PRACTICE

Find the first and second derivative of each of the functions in Exercises 1–8.

1. $f(x) = 3e^x$

2. $f(x) = -6e^x$

3. $f(x) = x^2 + 5e^x$

4. $f(x) = 4x^2 - 2e^x$

5. $f(x) = xe^x$

6. $f(x) = -7x^2e^x$

7. $f(x) = 9^x$

8. $f(x) = 3\cdot 5^{2x-1}$

For Exercises 9–18, find a formula for $f'(x)$ and determine the slope $f'(a)$ at the point where $x = a$ is given.

9. $f(x) = e^x \ln x;\ \ x = 1$

10. $f(x) = e^x \ln(x+4);\ \ x = 1$

11. $f(x) = \dfrac{e^x}{e^x - 1};\ \ x = 2$

12. $f(x) = \dfrac{e^x}{\ln x};\ \ x = e$

13. $f(x) = 2e^{4x};\ \ x = -1$

14. $f(x) = 8e^{-3x};\ \ x = 4$

15. $f(x) = 3e^{2x+1};\ \ x = 0$

16. $f(x) = 5e^{\frac{x}{2}};\ \ x = 0$

17. $f(x) = 10^{\frac{2}{x}};\ \ x = 2$

18. $f(x) = -x\cdot 3^{-x};\ \ x = 0$

For Exercises 19–22, find a formula for $f'(x)$ and use it to determine the intervals on which $f(x)$ is increasing or decreasing.

19. $f(x) = e^{1-x^2}$

20. $f(x) = e^{-0.04x^2}$

21. $f(x) = \left(e^{2x} - 4\right)^2$

22. $f(x) = \left(e^{4x} + 2\right)^3$

For Exercises 23 and 24, determine $f'(x)$ and use it to determine the intervals on which $f(x)$ is increasing or decreasing. Determine for each function if there is a horizontal asymptote. Confirm your results with a graphing calculator.

23. $f(x) = \sqrt{e^{-0.2x} + 11}$

24. $f(x) = \dfrac{1}{\sqrt{3e^x + 1}}$

Find $f'(x)$ and use it to argue whether or not there is an oblique asymptote for each of the functions in Exercises 25 and 26.

25. $f(x) = \ln\left(e^x + 1\right)$

26. $f(x) = \ln\sqrt{5 + e^{2x}}$

In Exercises 27–34, find the absolute extrema of the given function on the indicated interval.

27. $f(x) = xe^{2x};\ [-2, 1]$

28. $f(x) = xe^{\frac{x}{3}};\ [-4, 0]$

29. $f(x) = x^2e^{-x};\ [-1, 2]$

30. $f(x) = 2x^2e^{-x};\ [-2, 3]$

31. $f(x) = 5e^{1-x^2};\ [-2, 1]$

32. $f(x) = 3xe^{-x^2};\ [-2, 2]$

33. $f(x) = (2x + 3)e^{-0.2x};\ [1, 4]$

34. $f(x) = (4x - 1)e^{-0.5x};\ [0, 3]$

For each of the functions in Exercises 35–40, **a.** find any critical values, **b.** find any hypercritical values, **c.** find all intervals of concavity, and **d.** sketch the graph of the function. If available, confirm your results with a graphing calculator.

35. $f(x) = xe^{-0.4x}$

36. $f(x) = 2xe^{-0.5x}$

37. $f(x) = 4x^2e^{-x}$

38. $f(x) = 3x^2e^{-x}$

39. $f(x) = e^x + e^{-x}$

40. $f(x) = \dfrac{e^x}{x}$

APPLICATIONS

41. Revenue: The demand for a product is given by $D(x) = 140e^{-0.05x}$, where x is the number of units sold each week and $0 \le x \le 30$.

a. Find the number of units sold that will yield maximum revenue.

b. Find the price per unit that will yield maximum revenue.

42. Revenue: The demand equation for a certain product is given by $D(x) = 210e^{-0.025x}$, where x is the number of units sold each week and $0 \le x \le 60$.

a. Find the number of units sold that will yield the maximum revenue.

b. Find the price per unit that will yield maximum revenue.

43. Advertising: An automobile manufacturer is planning a television advertisement campaign to introduce a new model for their truck. It is estimated that $N(t) = 600\left(1 - e^{-0.02t}\right)$ people (in thousands) will have seen the advertisement after t days of advertising. How fast is N increasing at the end of 7 days?

44. Insect population: The mosquito population of a pool of water is estimated to be $P(t) = 400 + 1400e^{-0.3t}$, where t is the number of hours after the pool has been treated. Find the rate of change in the population at the end of 5 hours.

45. Bacterial population: The population of bacteria in an experimental culture is estimated by $N(t) = \dfrac{10{,}000}{1 + 9e^{-0.14t}}$, where t is the number of hours after the experiment begins. How fast is the population changing at the end of 5 hours?

46. Disease control: The elk herd in a national park has been infected by a contagious disease. The number of infected animals is estimated by $N(t)=\dfrac{600}{1+49e^{-0.36t}}$, where t is the number of days after the disease was discovered. How fast is the disease spreading after 4 days?

47. Suppose the value of the inventory of original Winchester rifles at Bill's Antique Firearms Company has increased according to the formula $r(t)=\dfrac{8500}{1+10e^{-0.6t}}$, where r is the average value (in dollars) of one of their rifles and t is the number of years since 2000.

a. What was the average value of a rifle in 2000? In 2005?
b. At what rate was r changing in 2005? In 2006?
c. If there is an inflection point for $r(t)$, locate it and explain its significance in the application.
d. When is the rate of increase of r at a maximum?

48. Suppose an advertising campaign for the sale of a new magazine, *Dungeons and Creeps*, causes sales to vary according to the formula $S(t)=8\left(1-0.3e^{-0.2t+1}\right)$, where S is monthly sales in thousands of magazines and t is time in months since the ad campaign started.

a. What were the monthly sales when the ad campaign started?
b. What was the rate at which sales were changing after 4 months into the campaign?
c. What are the long-term monthly sales expectations?

49. A research scientist determines that a mass of algae in a pond grows according to $A(t)=1+2te^{-0.5t}$, where A represents the mass-density of algae in the pond in suitable units and t is the time in months ($t = 0$ corresponds to April 1st).

a. What day of the year corresponds to a maximum A-value?
b. When does the rate of decline in algae reach its maximum?

50. A certain calculus student recalls information according to the formula $p=70e^{-0.6x}+30$, where p is the percentage of information retained after x weeks.

a. After 4 weeks, what percentage of a lesson is retained?
b. After 4 weeks, at what rate is the percentage changing?
c. What does the model predict a few months after the calculus course is over?

51. Inexpensive videos detailing the championship basketball season of Castle High School are sold locally by a civic club to raise money for next year's team. The total sales are given by $S=\dfrac{12{,}500}{1+15e^{-0.5x}}$, where S is the total number of videos sold after x weeks.

a. What are the total sales after 3 weeks?
b. What is the rate of change of sales after 3 weeks?
c. After about how many weeks will the total sales begin to level off?
d. When is the sales rate increasing fastest? Illustrate this point graphically.

WRITING & THINKING

52. **a.** Find the equation of the tangent line to $f(x) = 2e^{-x^2+1} + 4$ at the point where $x = 2$.

b. Discuss the advantages and disadvantages of using the tangent line to get values of $f(x)$ for $x \geq 2$ rather than the function itself.

53. Determine k in the equations that follow by finding $f'(0)$. Use logarithmic differentiation.

a. Let $f(x) = 10^x$. Determine the value of k in the formula $f'(x) = k \cdot 10^x$.

b. Let $y = f(x) = \pi^x$. Determine k in the formula $f'(x) = ky$.

54. In a psychological experiment, Calculus I students repeatedly took a test that asked them to find the derivatives of ten functions. The students' responses are modeled by $R(t) = 100\left(1 - 2^{-t}\right)$, where t is the number of times the test has been given and R is the percentage of students scoring 90% or better on the test. Here, $0 \leq t \leq 6$ and $0 \leq R \leq 100$.

a. Use a graphing calculator to sketch the graph of $R(t)$.

b. Interpret $R(2) = 75$.

c. Interpret $R'(2) = 17.3$.

d. Why is $R(3) \neq 75 + 17.3$?

TECHNOLOGY

Use a graphing calculator to graph $f(x)$ and $f'(x)$. Then locate all extrema and all inflection points, if any.

55. $f(x) = e^{-x^2} \ln\left(x^2 + 2\right)$

56. $f(x) = \ln \dfrac{2 + 3e^{-x}}{x + 2}$