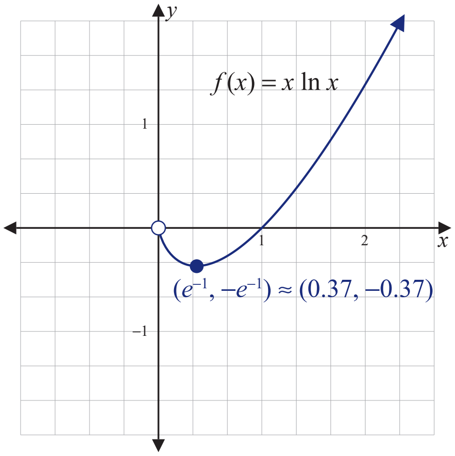
By testing values, we find that $f(x)$ is decreasing on the interval $(0, e^{-1})$ and increasing on the interval $(e^{-1}, +\infty)$.

Taking the second derivative of $f(x)$, we find that $f''(x) = \frac{1}{x} > 0$ for all x on the interval $(0, +\infty)$. Therefore, $f(x)$ is concave upward on the interval $(0, +\infty)$.

A local (and absolute) minimum occurs at the critical value $x = e^{-1}$, and

$$\begin{aligned} f\left(e^{-1}\right) &= e^{-1}\ln\left(e^{-1}\right) \\ &= e^{-1}(-1) = -e^{-1}. \end{aligned}$$

Note: One easy point to get without a calculator is (1, 0) (recall that ln 1 = 0). Therefore, it is possible to notice that $1 \cdot \ln 1 = 0$, and (1, 0) is an x-intercept.



5.3 EXERCISES

PRACTICE

Find the derivative of each of the functions in Exercises 1–32.

1. $f(x) = x^2 - \ln x$
2. $f(x) = 4x^2 + \ln\left(x^2\right)$
3. $f(x) = 25 + x\ln x$
4. $f(x) = \left(x^2+1\right)\ln x$
5. $y = \frac{\ln x}{x}$
6. $y = \frac{x^2}{\ln x}$
7. $y = (\ln x)^3$
8. $y = \sqrt{\ln x}$
9. $f(x) = \ln(5x+3)$
10. $f(x) = \ln(7x-2)$
11. $f(x) = \ln\left(x^2+2\right)$
12. $f(x) = \ln\left(x^2+3x\right)$
13. $f(x) = \ln\sqrt{2x^2-1}$
14. $f(x) = \ln\sqrt{x^3+4}$
15. $y = \ln\left((4x+3)^2\right)$
16. $f(x) = \ln\left(\left(x^2-4\right)^3\right)$
17. $y = \sqrt{x}\ln\left(x^2+2\right)$
18. $y = \frac{\ln(5x+2)}{x^3}$
19. $y = \frac{\ln\left(x^2+2x-1\right)}{x}$
20. $y = x^{-2}\ln\left(x^2-3x+4\right)$

21. $y = \log_3(9x)$

22. $f(x) = \log_2(7x^2 + 9)$

23. $y = 3x \log x$

24. $f(x) = \dfrac{\log_5(x)}{x^2}$

25. $f(x) = \ln\left((3x+1)(x^2+3)\right)$

26. $f(x) = \ln\left(x^2(4x-1)\right)$

27. $f(x) = \ln\left((2x-1)^2(x^2-2)\right)$

28. $f(x) = \ln\left(\sqrt{4x-7}(6x+7)\right)$

29. $y = \ln\left(\dfrac{x+1}{x-2}\right)$

30. $y = \ln\left(\dfrac{3x-1}{x+5}\right)$

31. $f(x) = \ln\left(\dfrac{x^2-5}{2x+9}\right)$

32. $f(x) = \ln\sqrt{\dfrac{4x+3}{x^2-6}}$

For Exercises 33–38, determine a formula for $f''(x)$. Use your calculator (if necessary) to solve the equation $f''(x) = 0$ and locate any possible inflection points.

33. $f(x) = \dfrac{x}{\ln x}$

34. $f(x) = (2x + x^2)\ln x$

35. $f(x) = 3x^2 \ln x$

36. $f(x) = \ln(x^3)$

37. $f(x) = (\ln x)^3$

38. $f(x) = \ln x + x^2 + 3x + 2$

Use logarithmic differentiation to find the derivatives of the functions in Exercises 39–46.

39. $y = (2x-5)^3\sqrt{x^2-2x}$

40. $y = (4-5x)^4(7x+2)^{-\frac{2}{3}}$

41. $y = (x^2+2)^4(x^2-1)^{-\frac{1}{3}}$

42. $y = (3x-2)^5\sqrt{x^2+x}$

43. $y = \dfrac{(2x+x^2)^3}{(4x-9)^2}$

44. $y = \dfrac{\sqrt[3]{x^2+4}}{(2-5x)^3}$

45. $y = \dfrac{(x+2)^2(3x+4)^2}{\sqrt{x-6}}$

46. $y = \dfrac{(x^2-3)^2\sqrt{x^2+3x}}{x+7}$

In Exercises 47–52, find the absolute extrema for each of the functions on the indicated interval.

47. $f(x) = x - \ln x;\ [0.5, 2]$

48. $f(x) = x^2 - 8\ln x;\ [0.3, 4]$

49. $f(x) = \dfrac{x}{\ln x};\ [1.2, 3]$

50. $f(x) = \dfrac{\ln x}{x^2};\ [1, 2]$

51. $f(x) = x^2 - \ln(x^3);\ [1, e]$

52. $f(x) = \ln(3 + 2x - x^2);\ [0, 2.5]$

Using the curve-sketching techniques discussed in Chapter 4, sketch the graph of each function in Exercises 53–58.

53. $f(x) = 4x\ln x$

54. $f(x) = x^2\ln x$

55. $f(x) = x - 3\ln x$

56. $f(x) = 4x - \ln(x^2)$

57. $f(x) = \dfrac{\ln x}{x}$

58. $f(x) = \dfrac{\ln x}{x^2}$

APPLICATIONS

59. Marginal revenue: A retailer has determined that the revenue from the sale of x end tables is given by the function $R(x) = 96x + \ln(4x^2 + 15)$ dollars. Find the marginal revenue.

60. Advertising: An automobile dealer has estimated that he can sell $N(x) = 420 + 72\ln(1 + 0.5x)$ cars annually, where x (in thousands of dollars) is the amount spent on advertising.

a. Find the number of cars sold if \$6000 is spent on advertising.

b. Find the rate of change in number of cars sold if \$6000 is spent on advertising.

61. Revenue: The daily demand for a product is given by $p = -8\ln(0.01x)$, where p is the price in dollars when x units are sold and $0 < x \le 100$. Find the maximum daily revenue.

62. Revenue: The demand for a product is given by $p = 14 - 6.5\ln x$, where x is the number of units (in thousands) that can be sold at a price p dollars and $2 \le x \le 8$. Find the maximum revenue.

63. Insect population: Mediterranean fruit flies are discovered in a citrus orchard. The Department of Agriculture has determined that the population of flies t hours after the orchard has been sprayed with pesticide is approximated by $N(t) = 25 - 5t\ln(0.04t) - t$, where $0 < t \le 25$.

a. Find $N(3)$, $N(10)$, $N(25)$.

b. What will be the maximum number of flies in the orchard?

64. Air quality: The Air Quality Management Board estimates that t hours after midnight in a major city the level of ozone in the air is about $N(t) = 0.013 - 0.007t\ln(0.026t)$ parts per million, where $0 < t \le 18$.

a. Find $N(6)$, $N(10)$, $N(18)$.

b. Find the maximum level of pollution.