So far, we have used the formula $A = Pe^{rt}$ to calculate the future value A of an account that will result from an investment P in the present. Now, suppose we know the future value to be accumulated and would like to determine how much should be deposited today to guarantee that future amount. We solve the formula $A = Pe^{rt}$ for P to obtain

$$P = Ae^{-rt}.$$

We call P the **present value** of A.

Example 4: Initial Deposit Amount

Suppose that the grandparents of a child would like to make a one-time investment now to help pay for the child's college education in 20 years. How much should they deposit in an account earning 10 percent interest compounded continuously, so that the value of the account will accumulate to \$20,000?

Solution

$$\begin{aligned} P &= Ae^{-rt} \\ &= 20{,}000e^{-0.10\cdot 20} && \text{We use } A = 20{,}000,\ r = 0.10, \text{ and } t = 20. \\ &\approx 20{,}000(0.135335283) \\ &\approx \$2706.71 \end{aligned}$$

That is, \$2706.71 (invested at 10 percent compounded continuously for 20 years) is the present value of \$20,000.

5.1 EXERCISES

PRACTICE

Use a calculator to evaluate the expressions in Exercises 1–8. Round your answers to the nearest hundredth.

1. $10^{2.3}$ **2.** $2^{1.5}$ **3.** $e^{-0.5}$ **4.** $e^{2.1}$

5. e^{3} **6.** e^{-1} **7.** $2e^{-1.6}$ **8.** $3e^{2.5}$

9. Let $f(x) = 9^{\frac{x}{2}}$.

a. Find $f(0)$. **b.** Find $f(1)$. **c.** Find $f(3)$.

10. For $y = -2e^{x^2+1}$, find the value of y (rounded to two decimal places) if

a. $x = 1$, **b.** $x = 0$, **c.** $x = -1$.

Graph each of the functions in Exercises 11–26.

11. $y = 4^x$ **12.** $y = 5^x$ **13.** $y = \left(\frac{2}{3}\right)^x$ **14.** $y = \left(\frac{1}{4}\right)^x$

15. $y = 3^{-2x}$ **16.** $y = 2^{-3x}$ **17.** $y = 2^{x-1}$ **18.** $y = 2^{x+1}$

19. $y = 3e^x$ **20.** $y = 2e^{-x}$ **21.** $y = e^{-1.5x}$ **22.** $y = e^{2x}$

23. $y = 6e^{-0.5x}$ **24.** $y = 10e^{0.5x}$ **25.** $y = e^{-x^2}$ **26.** $y = 5e^{-x^2}$

APPLICATIONS

27. Compound interest: Suppose that \$5000 is invested at an annual interest rate of 6 percent for 4 years. Find the total amount available if the interest is compounded:

a. annually, **b.** semiannually,
c. quarterly, **d.** daily (using 360 days/year),
e. continuously.

28. Compound interest: Suppose that \$3000 is deposited in a saving account at the rate of 8 percent per year. Find the total amount available at the end of 5 years if the interest is compounded:

a. annually, **b.** semiannually,
c. quarterly, **d.** daily (using 360 days/year),
e. continuously.

29. Compound interest: How much money must be deposited in a savings account that pays interest at a rate of 7 percent per year compounded quarterly so that after 5 years the balance will be \$12,000?

30. Compound interest: Matt's father is planning to open a savings account to help pay for Matt's college education. He has found a bank that will pay 8 percent interest compounded monthly. How much will he need to deposit initially so that in 4 years the balance will be \$20,000?

31. Compound interest: Maggie has \$8000 to invest for 2 years. She has two different accounts she can use. One account will pay 8 percent interest compounded monthly. The other account will pay 7.5 percent compounded continuously. Which is the better investment, and by how much?

32. Reliability: The reliability of a certain type of flashlight battery is given by $f(t) = e^{-0.05t}$, where f is the fraction of the batteries that last t hours. What fraction of the batteries produced are still working after 20 hours of use?

33. Drug concentration: The concentration of a certain drug in the body fluids is given by $C = C_0\, e^{-0.4t}$, where C_0 is the initial dose and t is the time in hours elapsed after the dose is administered. If 20 mg of the drug is given, how much of the drug will remain in the body after 8 hours?

34. Bee population: A colony of bees grows according to the formula $P = P_0 e^{0.18t}$, where P_0 is the number of bees present initially and t is the time in days. How many bees will be present after 6 days if there were 1200 bees initially?

35. Advertising: A radio station estimates that during an intense advertising campaign, the total number of people N who will hear a commercial is given by $N = A\,(1 - e^{-0.03t})$, where A is the number of people in the broadcasting area and t is the number of hours after the first time the commercial is run. If there are 500,000 people in the area, how many will hear the commercial during the first 20 hours of the ad campaign?

36. Reliability: Studies show that the fraction P of light bulbs that have burned out after t hours of use is given by $P = 1 - e^{-0.03t}$. What fraction of the bulbs have burned out after 24 hours?

37. Depreciation: The value V of a machine at the end of t years is given by $V = C(1 - r)^t$, where C is the original cost of the machine and r is the rate of depreciation. Find the value at the end of 6 years of a machine that costs \$8400 if $r = 0.15$.

38. Depreciation: The value V of a machine at the end of t years is given by $V = C(1 - r)^t$, where C is the original cost of the machine and r is the rate of depreciation. A machine that is 5 years old is valued at \$4000. If the rate of depreciation is 12 percent, find the original cost of the machine.

39. Present value: Mr. and Mrs. Jackson want to accumulate another \$50,000 as part of their retirement fund in 15 years with a one-time deposit now. If the interest is to be figured at 8 percent compounded continuously, what should the Jacksons' initial deposit be?

40. Present value: If a trust fund is designed to have a value of \$600,000 in 25 years and interest is compounded continuously at 9 percent, what is the present value of this amount?