4.4 EXERCISES

PRACTICE

For each of the rational functions in Exercises 1–12, find **a.** any vertical asymptotes, **b.** any horizontal asymptotes, and **c.** any oblique asymptotes.

1. $f(x)=\dfrac{1}{x-4}$ **2.** $f(x)=-\dfrac{3}{x+6}$ **3.** $f(x)=\dfrac{2x}{x+8}$

4. $f(x)=\dfrac{5x}{2x+1}$ **5.** $f(x)=\dfrac{x+2}{x^2+1}$ **6.** $f(x)=\dfrac{x-7}{x^2+3}$

7. $f(x)=\dfrac{5x^2}{3x^2-2x-1}$ **8.** $f(x)=\dfrac{2x^2}{x^2+3x}$ **9.** $f(x)=\dfrac{x^2-4}{x}$

10. $f(x)=\dfrac{3x^2+2}{x}$ **11.** $f(x)=\dfrac{x^2+1}{x+1}$ **12.** $f(x)=\dfrac{x^2-5}{x-2}$

In Exercises 13–22, sketch the graph of each rational function. Show any asymptotes on each graph.

13. $f(x)=-\dfrac{2}{x+5}$ **14.** $f(x)=\dfrac{4}{x-3}$ **15.** $f(x)=\dfrac{2x}{x+1}$

16. $f(x)=\dfrac{3x}{x-2}$ **17.** $f(x)=\dfrac{x-2}{x-1}$ **18.** $f(x)=\dfrac{x+4}{2x+1}$

19. $f(x)=2x+\dfrac{2}{x}$ **20.** $f(x)=3x+\dfrac{12}{x}$ **21.** $f(x)=\dfrac{3x^2+6}{x}$

22. $f(x)=\dfrac{2x^2+1}{3x}$

APPLICATIONS

23. Junker Renovation completely overhauls junked or abandoned cars. Data shows their 1970s models hold their value quite well. The value $F(x)$ of one of these cars is given by $F(x)=70-\dfrac{15x}{x+1}$ where x is the number of years since repurchase and F is in hundreds of dollars.

a. What is the initial resale price of a car?
b. Find all asymptotes.
c. Sketch the function.
d. What is the long-term value of one of these cars?

24. The average cost $A(x)$ is the total cost $C(x)$ divided by the quantity x. Thus $A(x)=\dfrac{C(x)}{x}$. If the total cost function for a product is $C(x)=3x+12$, graph the average cost $A(x)$. If there are any asymptotes, locate them and interpret their meaning.

25. A product's total costs are given by $C(x) = 0.03x^2 + 24x + 10$.

a. Graph the average cost function, locating any asymptotes.

b. What is the meaning of the asymptotes for average cost?

26. The Polar Pollution Control Company removes debris from old motors. Suppose the cost $F(x)$ of removing x percent of the pollutants is given by $F(x) = \dfrac{100,000}{100 - x}$, where x is a percentage, $0 \le x < 100$, and F is in dollars.

a. Determine $\lim\limits_{x \to 0^+} F(x)$ and $\lim\limits_{x \to 100^-} F(x)$ and interpret their meanings.

b. What percentage can be removed at a cost of \$3000?

c. Show that $F(x)$ is always increasing. Does this make sense in the context of the problem?

27. The cost of camel rides in Tunisia is modeled by the function $F(x) = 40 - \dfrac{20x}{x + 3}$, where x is the number of years since 2000 and F is a national average cost in dinars.

a. What was the cost of a camel ride in 2002?

b. What are the asymptotes for F and which is significant in the problem?

c. When was the average cost 26 dinars?

28. The sugar level concentration in the bloodstream of a certain diabetes patient is modeled by $S(t) = 1 + \dfrac{0.2t}{t^2 + 2}$, where S is in suitable units and t is the time in hours following a meal of allowed carbohydrate content.

a. Which asymptotes play a role here?

b. For $0 \le t \le 6$, what is the highest S-value and when does it occur? (If this level exceeds 4, the patient will become ill.)

c. Are there any inflection points? What is the meaning in the context of the problem of an inflection point?

29. Data suggests a professional football team will win $F(x)$ games (out of 17) if the salary of the superstar players increases. For one team, the function F is given by $F(x) = 8 + \dfrac{6x - 96}{0.125x^2 - 4x + 34}$, where x is the average salary (in millions) of the superstars (players earning at least one million dollars).

a. Are there any asymptotes of consequence in the problem?

b. What average salary gives the biggest return on games won, according to this model? (Here return is total games won.)

30. If administrative assistants at Bookworm Publications make phone follow-ups after textbook reviews, more colleges and universities will adopt a new statistics book. The publisher noticed that new book sales varied in the second year according to $S(x) = A_0 - \dfrac{(x + 200)}{x^2}$, where S is total sales and x is the number of phone calls made to colleges which have adopted the book. A_0 denotes the sales from the previous year.

a. Assuming $A_0 = 2500$, what sales can Bookworm Publications expect if they make 100 follow-up phone calls?

b. What is the horizontal asymptote and what is its significance?