We set $f''(x) = 0$ and obtain

$$16x - 12x^2 = 0, \text{ so}$$
$$4x(4 - 3x) = 0.$$

Thus $x = 0$ and $x = \frac{4}{3}$ will make $f''(x)$ equal to zero. Both values give inflection points for $f(x)$.

We can use $f'(x) = 0$ to locate the maximum and minimum points for $f(x)$.

$$f'(x) = 0$$
$$8x^2 - 4x^3 = 0$$
$$4x^2(2 - x) = 0$$
$$x = 0, 2$$

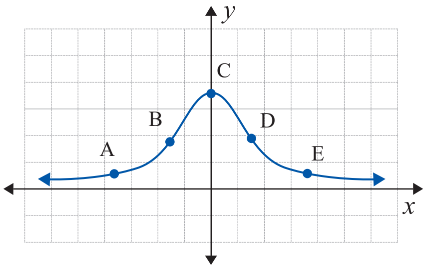
In Figure 8, we observe $x = 0$ corresponds to an inflection point (neither a maximum nor a minimum) and $x = 2$ corresponds to the maximum point $\left(2, \frac{31}{3}\right)$.

In this problem, the algebraic solutions of $f'(x) = 0$ and $f''(x) = 0$ are of a familiar type. When these equations are too difficult for an algebraic solution, a graphing calculator is invaluable.

4.1 EXERCISES

PRACTICE

1. At each point marked on the graph of f, determine if f' is positive, negative, or zero. Determine if f'' is positive, negative or zero.



Draw a graph that satisfies the given conditions in Exercises 2–5.

2. $f(5) = 9, \quad f'(5) = 2, \quad f''(5) = -2$

3. $f(-5) = -9, \quad f'(-5) = 2, \quad f''(-5) = 2$

4. $f(5) = -9, \quad f'(5) = 0, \quad f''(5) = 3$

5. $f(0) = 12, \quad f'(0) = 0, \quad f''(0) = -3$

For Exercises 6–13, find $f''(x)$. Then evaluate $f''(0)$, $f''(1)$, and $f''(4)$, if they exist.

6. $f(x) = x^3 + x^2 + 3$

7. $f(x) = x^3 - x^2 + 7$

8. $f(x) = x^2 - 5\sqrt{x} + 1$

9. $f(x) = x^2 + 2\sqrt{x} - 3$

10. $f(x)=\sqrt{x-4}$

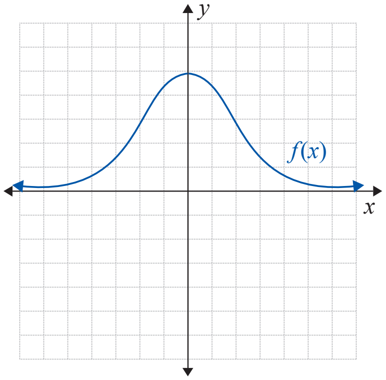
11. $f(x)=\sqrt{2x+1}$

12. $f(x)=\dfrac{x}{x+5}$

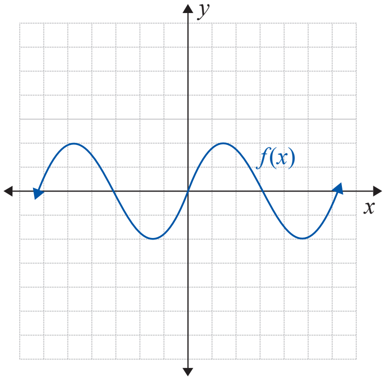
13. $f(x)=\dfrac{x-2}{x+4}$

In Exercises 14–17, sketch a possible graph for $f'(x)$ on the same coordinate axes as $f(x)$. Then locate all inflection points on the graph of $f(x)$.

14.



15.



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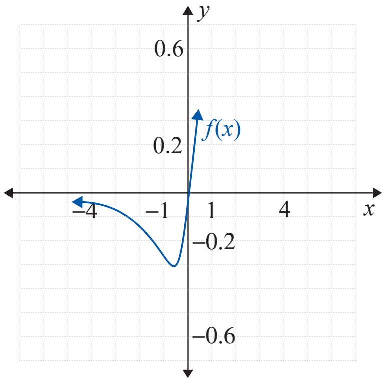

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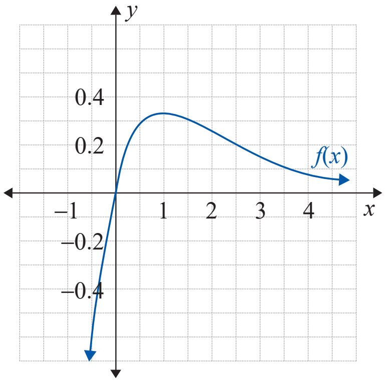
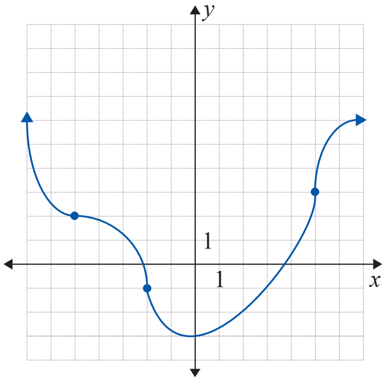


For each of the graphs in Exercises 18–21, list the interval(s) **a.** on which *f* is concave upward and **b.** on which *f* is concave downward; then **c.** locate all points of inflection.

18.



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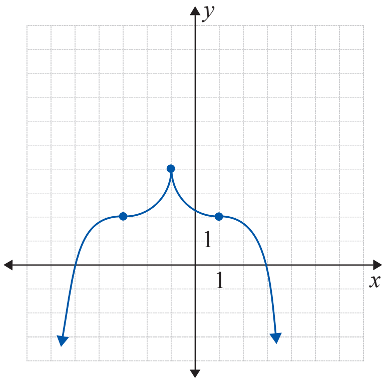

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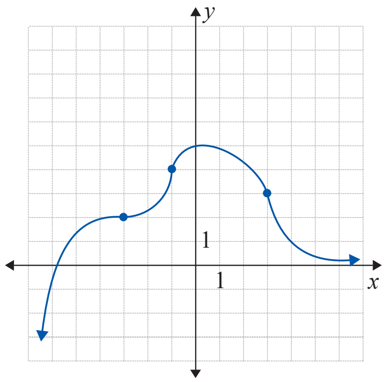
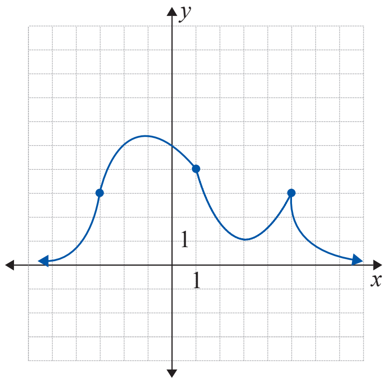


21.



In Exercises 22–33, determine the intervals on which each function is **a.** concave upward and **b.** concave downward; then **c.** locate all points of inflection. Use the information gathered to sketch the function. Confirm the details with a graphing calculator.

22. $f(x) = 2x^2 + 5x - 9$

23. $f(x) = 5x^2 + 8x - 1$

24. $f(x) = x^3 - 3x^2 + 7$

25. $f(x) = x^3 + 6x^2 - 10$

26. $f(x) = x^3 + 11x - 4$

27. $f(x) = 5x^3 + 7x + 2$

28. $f(x) = \frac{1}{3}x^3 - 2x^2 + x - 3$

29. $f(x) = \frac{1}{3}x^3 + 3x^2 + 2x - 5$

30. $f(x) = \sqrt[3]{2x+3}$

31. $f(x) = \sqrt[3]{5x-3}$

32. $f(x) = \frac{x}{x^2 - 4}$

33. $f(x) = \frac{4x}{x^2 - 5}$

WRITING & THINKING

In Exercises 34–37, give an example of a polynomial function that satisfies the conditions.

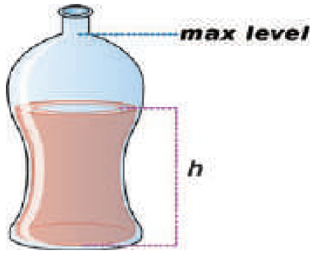
34. $F(5) = 15$; $F'(x)$ is nonzero, but $F''(x) = 0$ for all x.

35. $G(0) = 0$, $G'(0) = 0$, and $G''(0) = 0$; $G(x)$ is concave upward everywhere and has no inflection points.

36. $H(4) = 0$; $H'(x)$ is positive for $x > 4$ and negative for $x < 4$. $H(x)$ has no inflection points.

37. $J(4) = 0$; $J'(4)$ is zero but $J'(x)$ is positive if $x \neq 4$; $J''(4) = 0$.

APPLICATIONS



38. Filtrate: In a chemistry lab a filtrate drips slowly but continuously at a constant rate into a glass container shaped like the one shown. The container eventually fills to the base of the neck. Let t denote the passage of time and h be the height of the liquid.

a. Describe at what points on the bottle $\frac{dh}{dt}$ will be a maximum and a minimum.

b. Sketch a graph of $\frac{dh}{dt}$. Are there any inflection points on a graph of $y = h(t)$?

c. Add a sketch of $y = h(t)$ on the same coordinate axes as in part **b.**