## 3.5 EXERCISES

### PRACTICE

For each of the functions in Exercises 1–20, **a.** find the critical values, and **b.** use the First Derivative Test to find any local extrema.

1. $f(x) = 4x - x^2$
2. $f(x) = 9x - x^2$
3. $f(x) = x^2 + 6x - 2$
4. $f(x) = x^2 - 10x + 12$
5. $f(x) = \frac{1}{2}x^2 - 4x + 3$
6. $f(x) = -\frac{1}{2}x^2 + 3x - 2$
7. $f(x) = x^3 + x^2 - x + 3$
8. $f(x) = x^3 + 2x^2 + x - 2$
9. $f(x) = -x^3 - \frac{3}{2}x^2 + 18x + 6$
10. $f(x) = 2x^3 + x^2 - 4x$
11. $f(x) = x^3 - 3x + 6$
12. $f(x) = x^3 + 3x^2 - 4$
13. $f(x) = x + \frac{9}{x}$
14. $f(x) = x - \frac{4}{x}$
15. $f(x) = \frac{x^2 - 16}{x}$
16. $f(x) = \frac{4x^2 - 9}{x}$
17. $f(x) = 9x + x^{-1}$
18. $f(x) = 25x + x^{-1}$
19. $f(x) = 16x + x^{-2}$
20. $f(x) = 54x - x^{-2}$

### APPLICATIONS

21. **Rate of dictation:** It has been determined that after $t$ weeks of class, the average students in an intermediate shorthand class can take dictation at a rate of $W(t) = 60 + \frac{70t^2}{t^2 + 15}$ words per minute. Show that the rate of dictation is an increasing function which approches an upper bound.

22. **Court reporting:** A typical student in an intermediate court reporting class can reach a level of recording $W(t) = 40 + \frac{35t^2}{t^2 + 20}$ words per minute after $t$ hours of instruction and practice. Show that the number of words recorded per minute increases up to a certain level.

23. **Marathon running speed:** The speed at which a marathon runner travels varies over time. The function $F(t) = 5 + \frac{5t^2}{t^2 + 200}$ describes the velocity of a particular runner at time $t$ ($F(t)$ is in miles/hour). Determine the intervals over which the function is increasing or decreasing. Is this particular runner an experienced runner? (Experienced runners run faster near the end of the race.)

24. **Heating:** A frozen pizza is placed in the oven at $t = 0$. The function $F(t) = 30 + \frac{320t^2}{t^2 + 100}$ approximates the temperature of the pizza at time $t$. Show that the temperature approaches an upper bound. (The pizza will approach oven temperature over time.)

**25. Cooling:** A cup of hot coffee is placed in a room. The temperature in degrees Fahrenheit of the coffee is approximated by the function $F(t) = 180 - \frac{100t^2}{t^2 + 40}$ where $t$ is the number of minutes the coffee has been in the room. Show that the temperature function is a decreasing function, and find the temperature of the room. (The coffee approaches room temperature over time.)

**26. Velocity and acceleration:** The velocity of a car varies according to the function $F(t) = \frac{t^3}{1875} - \frac{119t^2}{1500} + \frac{53t}{15} + 5$, where $t$ is time ($0 < t < 100$). Determine any local maximum and minimum velocities, and the times at which they occur. (Be sure to specify whether each is a maximum or mininum.)

**27. Skydiving:** The velocity of a skydiver after parachute deployment is given by the formula $F(t) = 180 - \frac{165t^2}{t^2 + 10}$, where $t$ is the time after deployment. Show that the function is decreasing, and determine the terminal velocity for the diver-parachute system. (The velocity approaches terminal velocity as $t$ goes to infinity.)

**28. Pressure:** The pressure in a pressure cooker is given by the function $F(T) = 1 + \frac{3T^2}{T^2 + 180}$, where $T$ is the temperature inside the kettle. Show that the function is increasing, and determine the upper bound for pressure.

**29. Space probe closing speed:** In order to minimize travel time, a fictional probe on its way to a distant star accelerates for part of the voyage, and then decelerates to enter orbit safely. The closing speed of the probe with the star is given by the function $F(t) = \frac{t^2}{25} - 4t + 0.5$, where $t$ is the number of years after launch and $F(t)$ is measured in percentage of light speed. At what time does the probe begin decelerating? What is the probe's maximum closing speed? (**Hint:** Closing speed is negative.)

**30. Computation speed:** An experimental supercomputer is undergoing testing to determine whether it will meet the necessary specifications. The number of computations it can perform per second is found to be modeled by the function $F(t) = \frac{89t^3}{441{,}000} - \frac{757t^2}{22{,}050} + \frac{2227t}{1470}$, where $0 < t < 100$ is the number of minutes after startup and $F(t)$ is in quadrillions of computations/sec. Find all relative maximums and minimums for the function over the interval. Round to the nearest tenth. If the number of computations is zero at any time after startup, the computer crashes. Does the computer crash during testing?

**31. Fuel economy:** The fuel consumption of an automobile is not constant. Fuel economy depends largely on the speed of the vehicle. The function $F(v) = -\frac{8v^3}{19{,}125} + \frac{28v^2}{85} - \frac{288v}{85} + 80$ ($F(v)$ is in miles per gallon) describes the fuel consumption of a new hybrid vehicle, where $0 < v < 85$ is the velocity of the vehicle. On what intervals is the consumption increasing? Which velocities yield maximum efficiencies? (Remember, high efficiency means low consumption. Round to the nearest tenth.)