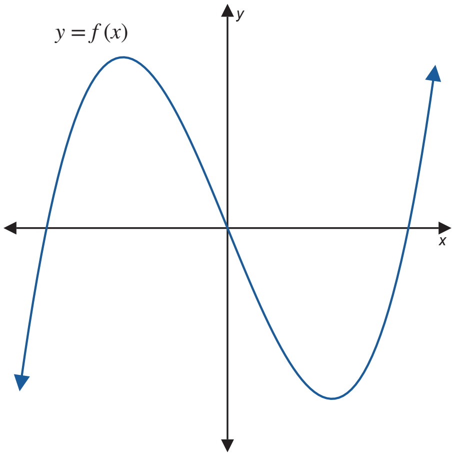
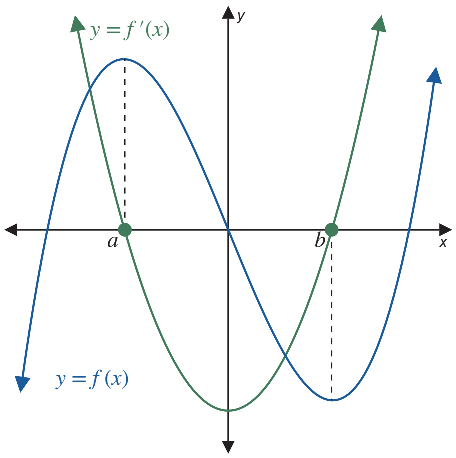
Example 5: Graphing $f'(x)$

The graph for $y = f(x)$ is given below. Identify the portions which show where $f'(x)$ is negative, positive, or zero. Draw a possible graph of $f'(x)$ by estimating the absolute values of the slopes on $f(x)$.



Solution



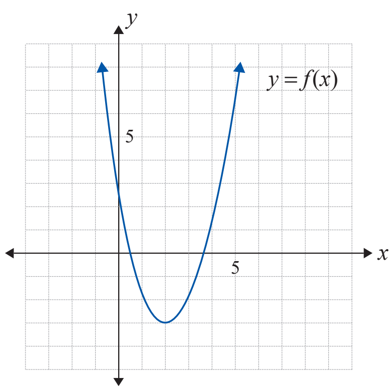
Let $x = a$ and $x = b$ (say $a < b$) denote the two x-values corresponding to the two local extremes. Then $f'(a) = f'(b) = 0$. In between a and b, the y-values decrease on the interval (a, b); therefore y' is negative. At $x = b$, the y-values start to increase (and y' becomes positive). Thus the graph of y' includes points $(a, 0)$ and $(b, 0)$ and lies below the x-axis (as y' is negative) from $x = a$ to $x = b$. For $x > b$ and $x < a$, the graph of y' lies above the x-axis. We draw a smooth curve from $(a, 0)$ to $(b, 0)$ and extend it in both directions above the x-axis. The low point for y' seems to occur near $x = 0$ because the graph of y is steepest near $(0, 0)$.

3.4 EXERCISES

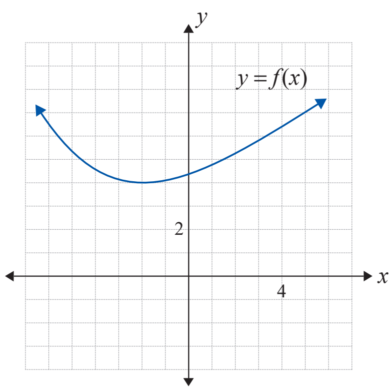
PRACTICE

In Exercises 1–10, find the open intervals on which **a.** f is increasing, and **b.** f is decreasing.

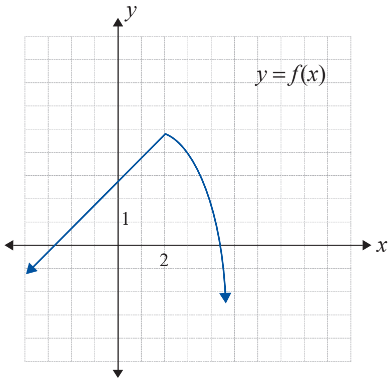
1.



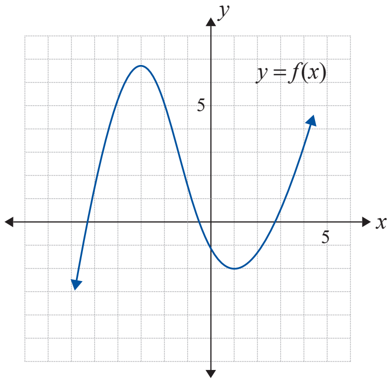
2.



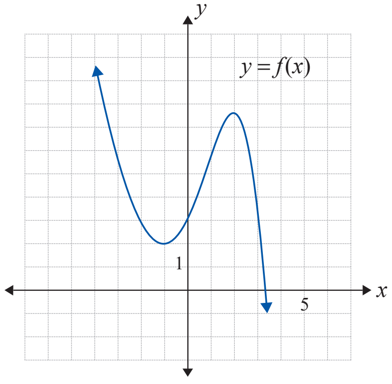
3.


y
y = f(x)
1
2
x

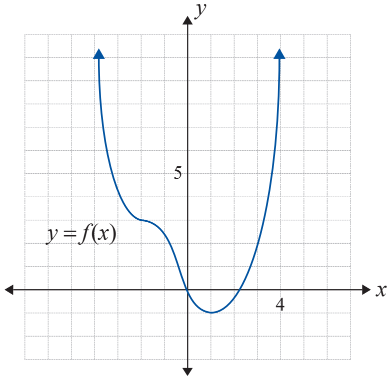
4.


y
y = f(x)
5
5
x

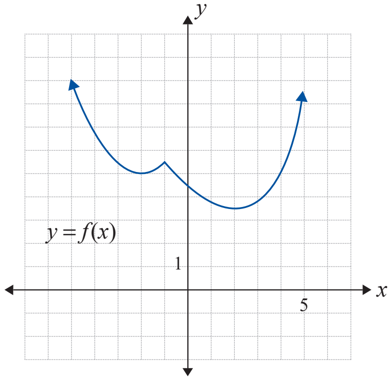
5.


y
y = f(x)
1
5
x

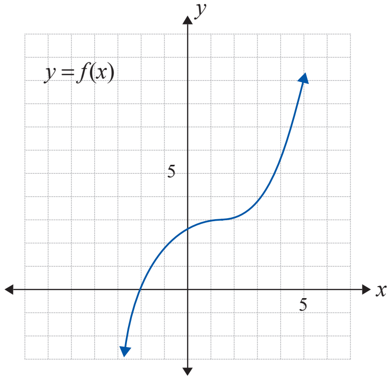
6.


y
5
y = f(x)
4
x

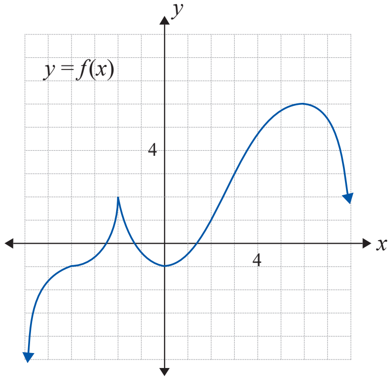
7.


y
y = f(x)
1
5
x

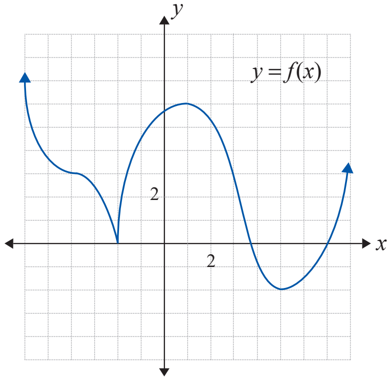
8.


y
y = f(x)
5
5
x

9.

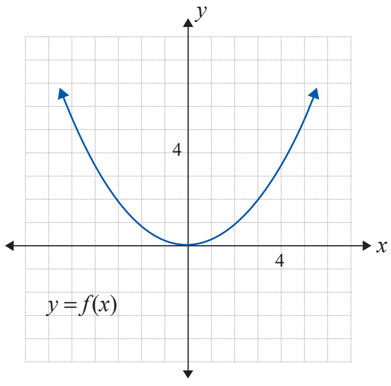

y
y = f(x)
4
4
x

10.

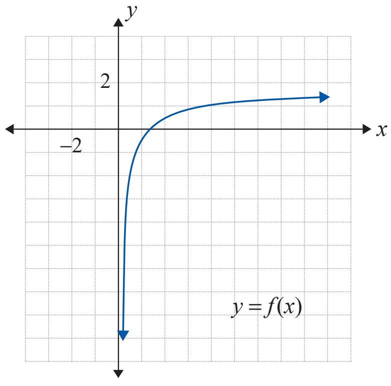

y
y = f(x)
2
2
x

For each of the graphs of $f(x)$ in Exercises 11–13, **a.** determine the open intervals on which the function is increasing and the open intervals on which it is decreasing, and **b.** sketch a possible graph of $f'(x)$.

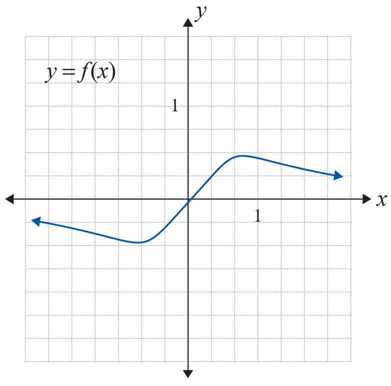
11.



12.



13.



For each of the functions in Exercises 14–33, **a.** find all values of x that correspond to horizontal tangent lines, **b.** find the open intervals on which the function is increasing and the open intervals on which it is decreasing, and **c.** graph the function.

14. $f(x) = x^2 - 8x + 3$

15. $f(x) = 2x^2 + 12x - 1$

16. $f(x) = 5 - 3x - x^2$

17. $f(x) = 7x - 2x^2$

18. $f(x) = 2 - 4x - 2x^2$

19. $f(x) = 3x^2 - 4x + 2$

20. $f(x) = (2x + 3)^2$

21. $f(x) = (3x - 2)^2$

22. $f(x) = 2x^3 - 5$

23. $f(x) = 3x^3 + 4$

24. $f(x) = x^3 - 3x^2 + 7$

25. $f(x) = x^3 - 6x^2 - 4$

26. $f(x) = x^3 - 3x^2 - 9x + 12$

27. $f(x) = x^3 - x^2 - x$

28. $f(x) = x^3 + \frac{1}{2}x^2 - 2x + 3$

29. $f(x) = x^3 - x^2 - 5x + 2$

30. $f(x) = \frac{1}{3}x^3 - x^2 - 8x + 10$

31. $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x - 6$

32. $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x - 6$

33. $f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + 4x + 11$

For each of the functions in Exercises 34–43, **a.** find all values of x that correspond to horizontal tangent lines, and **b.** find the open intervals on which the function is increasing and the open intervals on which it is decreasing.

34. $f(x)=\frac{x-1}{x}$

35. $f(x)=\frac{x}{x-2}$

36. $f(x)=\frac{x^2-4}{x}$

37. $f(x)=\frac{x^2-9}{x}$

38. $f(x)=\frac{x^3+16}{x}$

39. $f(x)=\frac{2x^3-27}{x^2}$

40. $f(x)=\frac{x+2}{x-1}$

41. $f(x)=\frac{x-5}{x+3}$

42. $f(x)=2x-\frac{125}{x^2}$

43. $f(x)=x^2+\frac{128}{x}$

APPLICATIONS

44. Revenue: A store manager has determined that the revenue from the sale of x units of a product is given by $R(x)=32x-0.4x^2$ dollars, where $0\le x\le 80$. On what interval of sales is the revenue increasing, and on what interval of sales is it decreasing?

45. Revenue: A producer of computer software has determined that the revenue from the production and sale of x units is given by $R(x)=48x-0.003x^2$ dollars, where $0\le x\le 10{,}000$. For what interval of production is the revenue increasing, and for what interval is it decreasing?

46. Profit: The revenue from the sale of x coffee makers is given by $R(x)=40x-0.4x^2$ dollars. The total cost is given by $C(x)=370+16x-0.2x^2$ dollars, where $0\le x\le 100$. Determine the interval(s) where the profit is increasing and where it is decreasing.

47. Profit: The revenue from the sale of x 50-gallon aquariums is given by $R(x)=54x-0.3x^2$ dollars. The total cost function is given by $C(x)=0.1x^2+4x+200$ dollars, where $0\le x\le 100$. Determine the interval of sales for which the profit is increasing and the interval for which it is decreasing.

48. Population: The population of the inner-city district of a city is given in thousands by $P(t)=24-0.3t+0.01t^2$, where t is the number of months after the implementation of an urban renewal project. How long will it be before the population starts to increase?

49. Wildlife management: In an attempt to naturally control the elk population in a national park, the U.S. Fish and Game Department has reintroduced the wolf into the area. It is estimated that the population of the elk herd will be $P(t)=600+12t-4t^{\frac{3}{2}}$, where t is the number of years after the reintroduction of the wolf. How long will it be before the elk population begins to decrease?

50. Average cost: The cost of producing x wireless speakers is given in dollars by $C(x)=320+30x+0.2x^2$, where $x\ge 0$. Determine the interval of production for which the average cost function is increasing.

51. Average cost: The cost of producing x units of a product is given in dollars by $C(x) = 250 + 45x - 0.2x^2$, where $x \geq 0$. Show that the average cost function is always decreasing. (This case corresponds to situations in which increased production distributes the cost so that the cost per unit decreases.)