

$$\begin{aligned}
 0 &= (16)^{\frac{1}{4}} \cdot \frac{3}{4} (81)^{-\frac{1}{4}} \frac{dy}{dt} + (81)^{\frac{3}{4}} \cdot \frac{1}{4} (16)^{-\frac{3}{4}} (4) \\
 0 &= \cancel{2} \cdot \frac{\cancel{3}}{2\cancel{4}} \cdot \frac{1}{\cancel{3}} \cdot \frac{dy}{dt} + 27 \cdot \frac{1}{\cancel{4}} \cdot \frac{1}{8} \cdot \cancel{4} \\
 0 &= \frac{1}{2} \cdot \frac{dy}{dt} + \frac{27}{8} \\
 -\left(\frac{1}{2} \cdot \frac{dy}{dt}\right) &= \frac{27}{8} \\
 \frac{dy}{dt} &= (-\cancel{2}) \frac{27}{\cancel{8}_4} \\
 \frac{dy}{dt} &= -\frac{27}{4} = -6.75
 \end{aligned}$$

Because the derivative is negative, the capital must **decrease** at a rate of  $\frac{27}{4} = 6.75$  units per month to keep the current level of production.

### 3.3 EXERCISES

#### 💡 PRACTICE

Use implicit differentiation to find  $\frac{dy}{dx}$  for each of the equations in Exercises 1–20.

1.  $2x^2 + y^2 = 4$
2.  $x^3 + y^3 = 5$
3.  $2x^3 + y^3 = 8$
4.  $x^2 - y^2 = 16$
5.  $\sqrt{x} + \sqrt{y} = 1$
6.  $x - \sqrt{y} = 2$
7.  $x^2y = 2$
8.  $xy^2 = -1$
9.  $x^2 + xy + y^2 = -1$
10.  $x^3 + 2xy - y^2 = 3$
11.  $4x^2 + 3xy + y^2 = 2x$
12.  $x^3 + y^3 = 3xy$
13.  $\frac{1}{x} + \frac{x}{y} = 2x$
14.  $x^2 + \frac{2x}{y} = \frac{1}{x^2}$
15.  $x^2 + \sqrt{xy} = 2y^2$
16.  $2y + \sqrt{xy} = 5x^2$
17.  $x^2y^2 + xy^3 = x^4$
18.  $x^3y + xy^3 = 3x^3$
19.  $x^2 + (y-2)^2 = 16$
20.  $x^2 + 4(y+3)^2 = 9$

In Exercises 21–30, use implicit differentiation to find  $\frac{dy}{dx}$  for the given equations; then find the slope of the tangent line at the given point.

- |  |   |
|--|---|
| 21. $4x^2 - 8y^3 = 24$ ; $(2, -1)$                 | 22. $3x^3 + 5y^2 + x = 1$ ; $(-1, 1)$           |
| 23. $x^2y - y^2 + 4x + 8 = 0$ ; $(1, 4)$           | 24. $5x^2 + xy^2 + 2x = 8$ ; $(-2, 2)$          |
| 25. $x^3 + 2xy - y^2 = 0$ ; $(3, -3)$              | 26. $4x^2 - 3xy + y^2 = 7$ ; $(2, 3)$           |
| 27. $\frac{1}{x} + \frac{x}{y^2} = 2x$ ; $(1, -1)$ | 28. $3x - \frac{2x}{y^2} + x^2 = 12$ ; $(3, 1)$ |
| 29. $2y + \sqrt{xy} = 5x^2$ ; $(2, 8)$             | 30. $x^2 - \sqrt{xy} = 2y^2 + 3x$ ; $(4, 1)$    |

In Exercises 31–40,  $x$  and  $y$  are functions of a third variable,  $t$ . Use implicit differentiation to find an expression for  $\frac{dy}{dt}$ .

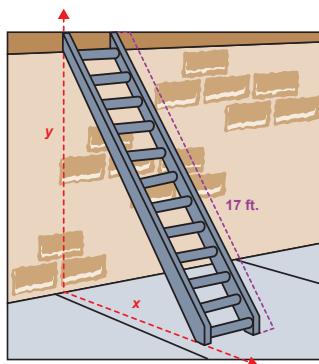
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|-----------------------------|-------------------------|-------------------------------|
| 31. $x^2 - 4y^2 = 16$       | 32. $3x^2 + y^4 = 4$    | 33. $x^3 + 5y^2 = 2x$         |
| 34. $6x^2 + 5x = 2y^2$      | 35. $\sqrt{xy} = 4$     | 36. $\sqrt{x} + \sqrt{y} = 3$ |
| 37. $x^2 + xy + y^2 = 3$    | 38. $x + 2xy - y^2 = 6$ | 39. $2xy + x^2 = y^3$         |
| 40. $x^2y^2 - y^3 + 4x = 0$ |                         |                               |

### APPLICATIONS

41. **Retail sales:** The manager of an audio electronics store has determined that the number of stereo receivers and the number of speaker systems sold weekly are related by the equation  $0.9y^2 = 10x + xy$ , where  $x$  is the number of receivers and  $y$  is the number of speaker systems. Find  $\frac{dy}{dx}$  if  $x = 12$  and  $y = 20$ , and interpret your answer.
42. **Retail sales:** The number of pairs of trousers  $x$  and the number of shirts  $y$  sold at a department store are related by the equation  $36x = 11y + 0.01x^2y$ . Find  $\frac{dy}{dx}$  when  $x = 10$  and  $y = 30$ , and interpret your answer.
43. **Cobb-Douglas production:** The level of production of a company is given by  $P = 30x^{\frac{1}{3}}y^{\frac{2}{3}}$  units monthly, where  $x$  is the units of labor and  $y$  is the units of capital. The company is currently utilizing 64 units of labor and 27 units of capital. If labor is increased by 2 units per week, what will be the change in units of capital per week to maintain the current level of production?
44. **Cobb-Douglas production:** The level of production of a company is given by  $P = 18x^{0.3}y^{0.7}$  units monthly, where  $x$  is the units of labor and  $y$  is the units of capital. The company is currently utilizing 35 units of labor and 24 units of capital. If capital is increased by 3 units per week, what will be the change in units of labor per week to maintain the current level of production?

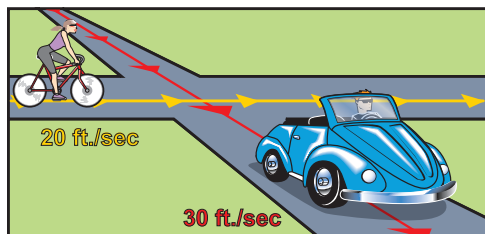
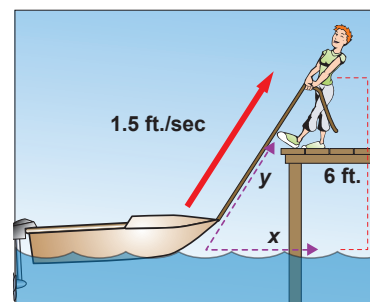
45. **Rate of increase in cost:** The cost of producing  $x$  units of a product is given by the function  $C(x) = 0.02x^3 - x^2 + 8x + 200$ . The factory is currently producing 60 units per week but plans to increase production at a rate of 3 units per week. What will be the rate of increase in the total cost?

46. **Rate of decrease in cost:** The cost of producing  $x$  units of a commodity is given by the function  $C(x) = x^2 - 2x^{\frac{3}{2}} + 7x + 180$ . Currently, the production level is 36 units per day. The company plans to decrease production at a rate of 2 units per day. What will be the rate of decrease in the total cost?



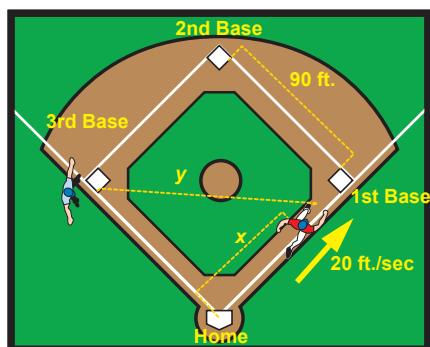
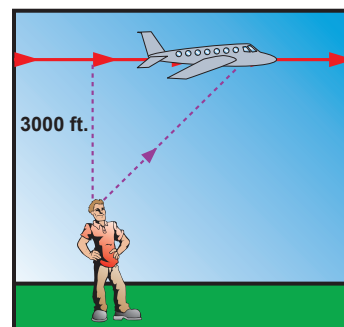
47. **Sliding ladder:** A 17 ft ladder is leaning against a wall. The bottom of the ladder is pulled away from the wall at a rate of 3 ft/sec. How fast is the top of the ladder moving down the wall when the top is 8 feet above the ground?

48. **Velocity:** Marijean is standing on a boat dock pulling in her boat by means of a rope attached to a boat at water level. Her hands are 6 feet above the water and she is pulling in the rope at a rate of 1.5 ft/sec. How fast is the boat approaching the dock if there are 10 feet of rope still out?



49. **Driving:** A car traveling south at 30 ft/sec crosses an intersection. When the car is 90 feet past the intersection, a bicyclist crosses the intersection traveling east at a rate of 20 ft/sec. How fast is the distance between the car and the bicycle increasing 5 seconds after the bicycle crosses the intersection?

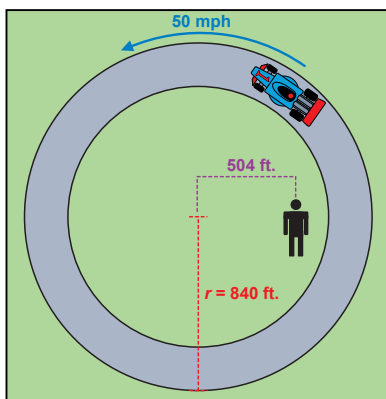
50. **Distance to an airplane:** An airplane traveling at a height of 3000 feet crosses directly over an observer. The speed of the plane is 400 ft/sec. How fast is the distance between the observer and the plane changing after 10 seconds?



51. **Baseball:** A base runner heads towards first base with a speed of 20 ft/sec. A baseball diamond is a square, 90 feet on each side.

- How long will it take for the runner to reach first base?
- What is the runner's rate of change of distance from the umpire standing on third base (we will call this  $\frac{dy}{dt}$ )? (**Hint:** Use the diagram to get a relation between  $x$  and  $y$ .)
- What is the runner's speed when he arrives at first base?

- 52. Sailing:** A Coast Guard radar monitoring station on shore observed a sailboat on its radar grid. It was determined that the boat's east-west distance along the shoreline changed by the formula  $x = 12 + 0.1t$  and its seaward distance (north-south) changed by the formula  $y = 20 - 0.3t$ . Here,  $x$  and  $y$  are in miles and  $t$  is in minutes.
- What was the sailboat's position at  $t = 0$  minutes? Sketch this situation.
  - Determine  $\frac{dx}{dt}$  and  $\frac{dy}{dt}$ .
  - What is  $\frac{dy}{dx}$  at  $t = 10$ ? Interpret this number.
  - Where and when will the sailboat hit the shore (assuming it keeps its present heading)?
- 53. Bags of oranges:** Suppose the wholesale price in Miami of bags of oranges satisfies a demand equation of  $xp + 20p = 1040$ , where  $x$  is the number of bags supplied and  $p$  is the demand (unit price) in dollars.
- If 500 bags are available today, what is the unit price?
  - If the supply is increasing at the rate of 100 bags per day, at what rate is the price changing?
- 54. Dripping water:** The radius of a pan of water is 7 cm and water from a tap drips in at the rate of 10 cubic centimeters per minute.
- Determine  $\frac{dh}{dt}$ , where  $h$  is the height of water in the pan.
  - How long will it take to fill the pan to a height of 7 cm?
- 55. Demand:** The Arrow Marketing Group sells teddy bears with college logos to college and university gift shops. Their demand equation is  $px - 2800 = x$ , where  $x$  is the quantity of bears and  $p$  is the price in dollars that each sells for.
- How many bears can be sold (or "demanded") at \$4.50 apiece?
  - Determine a formula  $\frac{dx}{dp}$  and evaluate for  $p$  and  $x$  as in part a.
- 56. Electricity costs:** Electricity costs per semester at Mount State University are calculated by the formula  $C = 0.05x + 0.03y - 0.08xy$ , where  $x$  is a measure of student size and activity,  $y$  is dependent on the usage of various buildings and their efficiencies, and  $C$  is in millions of dollars.
- Determine  $C$  if  $x = 7$  and  $y = 9$ .
  - Determine a general formula for  $\frac{dy}{dx}$  and evaluate for  $x = 7$  and  $y = 9$ .



- 57. Race track:** A circular race track has a radius of 840 feet. At a certain point in time,  $t_0$ , an observer in a maintenance pit 504 feet from the center of the track clocks a car (when it is directly north of him) traveling counterclockwise along the track at a speed of 50 miles per hour from his right to left  $\left(\frac{dx}{dt}\right)$ . (Use 1 mile = 5280 feet.)
- Determine the location of the car on the track. Assume the center of the track is at the origin.
  - Determine a general equation for  $\frac{dy}{dx}$ .
  - What is  $\left.\frac{dy}{dx}\right|_{t=t_0}$ ? Interpret its meaning.
  - What is  $\frac{dy}{dt}$  at  $t = t_0$ ?

- 58. Chlorination costs:** Chlorination costs for the swimming pool at a spa are given by  $C = 0.27x + 2y - 0.001xy^2$ , where  $x$  is the number of weekly swimmers,  $y$  is the number of special functions, and  $C$  is in dollars.
- Determine  $C$  if  $x = 500$  and  $y = 4$ .
  - Determine a formula for  $\frac{dy}{dx}$  and evaluate at  $x = 500$  and  $y = 4$ .
- 59. Probability measurement:** The standard deviation,  $S$ , of a binomial random variable is given by  $S = \sqrt{np(1-p)}$ , where  $n$  is the number of trials or repetitions of an experiment and  $p$  is the probability of success on one outcome.  $S$  is a measure of how the data tends to vary from the center (or mean).
- For  $n = 768$  and  $p = 0.25$ , determine  $S$ .
  - Suppose we consider  $S$  as a fixed quantity. Determine  $\frac{dp}{dn}$  for  $n$  and  $p$  as in part a.