

To find how fast this measure was changing in 1988 (8 years since 1980), evaluate the derivative for $t = 8$.

$$P'(8) = \frac{8^3 + 72 \cdot 8}{(8^2 + 36)^{\frac{3}{2}}} = \frac{512 + 576}{(100)^{\frac{3}{2}}} = \frac{1088}{1000} = 1.088$$

In 1988, the air pollution measure was increasing at a rate of 1.088 units per year.

3.2 EXERCISES

PRACTICE

Find the derivative for each function given in Exercises 1–40 and simplify your answer.

1. $f(x) = (2x - 5)^4$

2. $f(x) = (7x + 2)^3$

3. $f(x) = (1 - 4x)^3$

4. $f(x) = (3 - 5x)^5$

5. $g(x) = (x^2 + 4)^{-2}$

6. $g(x) = (x^2 - 8)^{-1}$

7. $h(t) = (2t^2 + 3t)^{-3}$

8. $h(t) = (4t^2 - t)^{-2}$

9. $y = (2x^2 + 5x - 7)^2$

10. $y = (4x^2 + 9x - 3)^3$

11. $y = (x^3 + 1)^{\frac{1}{2}}$

12. $y = (2x^3 - 5)^{\frac{1}{3}}$

13. $y = \sqrt[3]{4x^2 + 1}$

14. $y = \sqrt{7 + 4x^2}$

15. $y = \sqrt[4]{1 - 2x^3}$

16. $y = \sqrt[3]{5x^3 - 4}$

17. $f(t) = 5t(t^3 + 3)^4$

18. $f(x) = -7x(x^4 - 2)^3$

19. $f(x) = 2x^3(x^2 - 8)^3$

20. $f(t) = t(4 - 3t^2)^2$

21. $g(x) = \frac{1}{\sqrt{x^2 - 6}}$

22. $g(x) = \frac{5}{\sqrt{x^3 + 4}}$

23. $g(t) = \frac{t}{\sqrt{t^2 + 8}}$

24. $g(x) = \frac{x^2}{\sqrt[3]{x^2 + 6}}$

25. $h(x) = \frac{\sqrt[3]{2x + 3}}{x^2}$

26. $h(x) = \frac{\sqrt{5x - 2}}{x^3}$

27. $y = (2x + 1)\sqrt{3x - 4}$

28. $y = (4x + 3)\sqrt{x^2 + 3}$

29. $y = (3x - 2)^2(5x + 1)^{-2}$

30. $y = (2x + 7)^3(3x + 1)^{-4}$

31. $f(t) = \frac{5t + 1}{(t - 1)^{\frac{2}{3}}}$

32. $f(t) = \frac{t^2 + t + 1}{\sqrt{t^4 - 1}}$

33. $g(t) = \left(\frac{2t + 5}{t + 1}\right)^3$

34. $g(t) = \left(\frac{5t + 4}{t^2 - 3}\right)^4$

35. $y = \left(\frac{x + 3}{4 - 2x}\right)^{\frac{1}{2}}$

36. $y = \left(\frac{x^2}{4x + 1}\right)^{\frac{1}{3}}$

37. $y = \sqrt{\frac{x + 2}{3x - 1}}$

38. $y = \sqrt{\frac{x^2 + 6}{x^3}}$

39. $y = \frac{x^2 + x}{\sqrt{7-2x}}$

40. $y = \frac{(x^2 + 2)^2}{\sqrt{5x-3}}$

In Exercises 41–50, find $\frac{dy}{du}$, $\frac{du}{dx}$, and $\frac{dy}{dx}$. Then evaluate $\frac{dy}{dx}$ for the given value of x .

41. $y = u^2 + 2$, $u = 3x^2 + 1$; $x = -1$

42. $y = \sqrt{u+4}$, $u = x^2 + x - 1$; $x = 2$

43. $y = \frac{1}{u^2}$, $u = 2x^3 - 3x + 3$; $x = 1$

44. $y = \sqrt[3]{u}$, $u = 2x^3 - 4x$; $x = 2$

45. $y = u^{\frac{3}{2}}$, $u = x^3 - 2x^2$; $x = 3$

46. $y = \frac{1}{u^3}$, $u = 3x + 1$; $x = 1$

47. $y = \sqrt[3]{u}$, $u = 7x^2 + 1$; $x = -3$

48. $y = u^2 + 3u + 4$, $u = x^3 - 5x - 2$; $x = -2$

49. $y = 2u^2 - 5u + 3$, $u = 5x + 6$; $x = 2$

50. $y = 2u^3 - 3u + 1$, $u = x^3 + 8$; $x = 1$

In Exercises 51–56, use the given information to find $h'(2)$: $f(2) = 3$, $f'(2) = -1$, $g(2) = 4$, $g'(2) = 10$, $g(3) = 8$, and $g'(3) = 7$.

51. $h(x) = (f(x) + 1)^3$

52. $h(x) = \left(\frac{f(x)}{g(x)}\right)^2$

53. $h(x) = g(f(x))$

54. $h(x) = \sqrt{f(x) + g(x)}$

55. $h(x) = (g(x))^3$

56. $h(x) = (2 + 3 \cdot f(x))(g(x))^3$

Determine the equation of the tangent line for $f(x)$ at the x -value indicated in Exercises 57–60.

57. $f(x) = (3x - 1)^3$; $x = 1$

58. $f(x) = \left(\frac{x^2 + 1}{x + 1}\right)^3$; $x = 2$

59. $f(x) = (3x^2 + 2x + 8)^{\frac{1}{2}}$; $x = 4$

60. $f(x) = \sqrt{10x + 1}$; $x = 8$

APPLICATIONS

61. Marginal revenue: A dealer of microwave ovens estimates that he can sell x ovens per month when the demand function (price) is $p = D(x) = 20\sqrt{280 - 4x}$ dollars.

- Find the revenue function $R(x)$.
- Find $R(21)$.
- Find the marginal revenue function $R'(x)$.
- Find $R'(21)$.

- 62. Marginal revenue:** The demand function (price) for a particular product is given by $D(x) = 8\sqrt{25 - 5x + 0.25x^2}$ dollars, where x is the number of units (in hundreds) sold.
- Find the revenue function $R(x)$.
 - Find $R(2)$.
 - Find the marginal revenue function $R'(x)$.
 - Find $R'(2)$.
- 63. Population growth:** It is estimated that t years from now the population of Castle City will be $P(t) = 10(40 + 2t)^2 - 1600t$.
- What will the population be in 8 years?
 - Find the rate of change in population in 8 years.
- 64. Air pollution:** It is estimated that t years from now the level of air pollution in Bohrberg will be $P(t) = \frac{0.6\sqrt{8t^2 + 11t + 60}}{(t+1)^2}$ parts per million. Find the rate of change in the pollution level in 7 years.
- 65. Pollution:** After a sewage spill, the level of pollution in San Remo Bay is estimated by $P(t) = \frac{200t^2}{\sqrt{t^2 + 11}}$, where t is the time in days since the spill occurred. How fast is the level changing after 5 days? Round to the nearest whole number.
- 66. Bacterial growth:** It is estimated that in t hours the population of bacteria in a culture will be $P(t) = \frac{8000}{\sqrt{8 - 0.5t}}$.
- What will be the population in 8 hours?
 - Find the rate of change in the population in 8 hours.
- 67. Rate of change of cost:** A manufacturer of vacuum cleaners estimates that the total cost of producing x vacuum cleaners is given by $C(x) = -0.5x^2 + 56x + 800$ dollars. Records show that after t hours on a typical day, the number of units produced is given by $x = 5\sqrt{t^2 + 5t}$. Find the rate of change of total cost with respect to time at the end of
- 4 hours.
 - 5 hours.
- 68. Rate of change of profit:** A manufacturer has determined that the weekly profit from the sale of x items is given by $P(x) = -x^2 + 280x - 4000$ dollars. It is estimated that after t days in any week, $x = 0.5t^2 + 5t$ items will have been produced. Find the rate of change of profit with respect to time at the end of
- 4 days.
 - 5 days.

- 69. Pollution:** Studies show that the average level of certain pollutants in the air is given by $L = 1 + 0.2x + 0.001x^2$ parts per million when the population is x thousand people. It is estimated that t years from now the population will be $x = \frac{200}{\sqrt{7 - 0.5t}}$ in thousands. Find the rate of change of the level of pollutants after
- 6 years.
 - 12 years.
- 70. Security costs:** The annual cost for campus security is given by $C(x) = 3x^2 - 32x + 16$ in thousands of dollars. It is estimated that the enrollment in t years will be $x = 16 + 0.5t + 0.02t^2$ in thousands. Find the rate of change in security costs after
- 3 years.
 - 4 years.
- 71. Baseball attendance:** The average home attendance per week at a Class AA baseball park varied according to the formula $N(t) = (3 + 0.2t)^{\frac{1}{2}}$, where t is the number of weeks into the season ($0 \leq t \leq 12$) and N is in thousands of persons.
- What was the attendance during the first week into the season?
 - Determine the number $N'(5)$.
 - Interpret the meaning of $N'(5)$.
- 72. Weekly attendance:** The semester after its student team won an intercollegiate Duplicate Bridge championship, the average weekly attendance at the University Union Building varied according to the formula $B(t) = 100 - 50\left(1 - \frac{t}{16}\right)^{\frac{3}{2}}$, where t is the number of weeks after the championship ($0 \leq t \leq 16$) and B is the number of persons.
- What is $B(0)$ and what does it represent?
 - Determine $B'(t)$.
 - What is $B'(7)$? Interpret the meaning of this number.
- 73. Class registration:** The annual registration in university calculus classes varies according to the formula $P(t) = 1 + \left(1 - \frac{t}{30}\right)^{2.5}$, where t is the number of years since 2010 and P is in millions of students.
- Determine $P(0)$ and explain its meaning.
 - Determine $P'(t)$.
 - Calculate $P'(10)$ and explain its meaning.
- 74. Travel:** The number of passengers traveling from California to Central America and back on cruise ships is given by $C(t) = (10t + 50)^{1.5}$, where t is the number of years since 2005 and C is passenger count in thousands.
- When did the number of passengers hit 1,000,000 people ($C = 1000$)?
 - Compute $C'(t)$.
 - Calculate $C'(12)$ and interpret its meaning.