CAUTION

If the rules of differentiation have been followed, then an answer may be correct even though it does not "look like" the answer given. Be sure to check with your instructor to see how much algebraic simplification is expected and whether or not one form is preferred over another.

3.1 EXERCISES

PRACTICE

In Exercises 1–5, find $f'(x)$ two ways: (1) multiply the factors first, then find the derivative, and (2) use the Product Rule.

1. $f(x)=x^2(1+3x-2x^2)$

2. $f(x)=(x+3)(x-1)$

3. $f(x)=x^{\frac{1}{2}}(1+3x^2)$

4. $f(x)=x^{\frac{1}{2}}\left(1+x^{\frac{1}{2}}-x^{\frac{3}{2}}\right)$

5. $f(x)=(2x+3)(2x-3)$

In Exercises 6–10, find $g'(x)$ two ways: (1) divide the factors first, then find the derivative, and (2) use the Quotient Rule and simplify the answer.

6. $g(x)=\dfrac{1+5x+x^2}{x}$

7. $g(x)=\dfrac{2+\sqrt{x}}{\sqrt{x}}$

8. $g(x)=\dfrac{x^2+1}{x^5}$

9. $g(x)=\dfrac{30x^2-10x^6}{5x}$

10. $g(x)=\dfrac{3x^{\frac{1}{2}}-5x^{\frac{3}{2}}+7x^{\frac{5}{2}}-9x^{\frac{7}{2}}}{x^{\frac{1}{2}}}$

In Exercises 11–34, use the Product Rule or Quotient Rule to find the derivative of each of the functions. Simplify your answers.

11. $f(x)=x^3(x^2+5)$

12. $f(x)=x^5(2x-x^3)$

13. $f(t)=t^{\frac{1}{2}}(4t+3)$

14. $f(t)=t^{\frac{2}{3}}(4t^2+1)$

15. $y=x^2\left(\sqrt{x}+\dfrac{1}{\sqrt{x}}\right)$

16. $y=x^{-2}\left(3x+x^{\frac{1}{3}}\right)$

17. $g(u)=(2u^2+3)(5-3u)$

18. $g(u)=(3u^2-8)(u^2+u)$

19. $g(t)=\left(5+\dfrac{1}{t}\right)\left(t^2+\dfrac{1}{5}\right)$

20. $f(t)=\left(1-\dfrac{3}{t^2}\right)(2t^2+t-1)$

21. $f(x)=\dfrac{3x}{x+6}$

22. $f(x)=\dfrac{7x^2}{2x-1}$

23. $f(x)=\dfrac{x+8}{x-7}$

24. $f(x)=\dfrac{x^2+2x-3}{x+2}$

25. $y=\dfrac{x^3-5}{x^2+1}$

26. $y=\dfrac{2x^2+3x}{x^3+6}$

27. $g(x)=\dfrac{\sqrt{x}}{x+9}$

28. $g(x)=\dfrac{6\sqrt{x}}{3x-4}$

29. $f(u)=\dfrac{u^2}{\sqrt{u}+1}$

30. $f(u)=\dfrac{7}{1-\sqrt[3]{u}}$

31. $f(t)=\dfrac{4-\sqrt{t}}{t^2+3}$

32. $f(t)=\dfrac{3-t}{4-5\sqrt{t}}$

33. $f(x)=\dfrac{x^2-5x}{1+2\sqrt[3]{x}}$

34. $f(x)=\dfrac{x\left(1+3\sqrt{x}\right)}{\sqrt{x}+6}$

In Exercises 35–44, you are given that $f(x)$ and $g(x)$ are differentiable functions and that $f(2)=3$, $f'(2)=-1$, $g(2)=-11$, and $g'(2)=6$. In each exercise, find the value of $h'(2)$.

35. $h(x)=x\cdot f(x)$

36. $h(x)=\dfrac{f(x)}{2x+1}$

37. $h(x)=\dfrac{f(x)+3x}{f(x)-3x}$

38. $h(x)=\dfrac{g(x)}{f(x)}$

39. $h(x)=\dfrac{g(x)}{3x+10}$

40. $h(x)=(3x+5)\cdot f(x)$

41. $h(x)=\dfrac{16x+1}{f(x)-11x+1}$

42. $h(x)=f(x)\cdot g(x)$

43. $h(x)=\dfrac{f(x)}{g(x)}$

44. $h(x)=g(x)\cdot(1+3x)$

In Exercises 45–50, find the equation of the line tangent to the graph of $f(x)$ at the given point.

45. $f(x)=\left(x+5x^{\frac{1}{2}}\right)\left(6x^2-12x+2\right);\ (4, 700)$

46. $f(x)=\dfrac{\left(11x^2-3x+2\right)}{x^2+1};\ (1, 5)$

47. $f(x)=\dfrac{2-3x}{5+2x};\ (0, 0.4)$

48. $f(x)=\left(x^5-5\right)\left(x^3-x-1\right);\ (0, 5)$

49. $f(x)=\dfrac{20}{17x+3};\ (1, 1)$

50. $f(x)=\dfrac{\sqrt{x}+2}{x^2-1};\ \left(9, \dfrac{1}{16}\right)$

51. Given $f(x)=(1-x)\left(16-x^2\right)$, find the (x, y)-coordinates on the graph where the tangent line is horizontal.

52. Given $g(x)=(x-10)\left(x^2+2x+1\right)$, find any (x, y)-coordinates on $g(x)$ for which the tangent line is horizontal.

53. Find any point or points on the graph of $y=(x-5)(x+10)$ so that the slope equals 25. Sketch a graph of y and the tangent line or lines.

54. Find any point or points on the graph of $G(x)=(2x+1)(x-3)$ so that the slope is -20. Sketch a graph of G and the tangent line or lines.

55. Sketch a graph of $F(x)=\frac{30x}{2x^2+5}$ on the x-interval $[-5, 10]$. Determine the (x, y)-coordinates of any point with a horizontal tangent line, and sketch this (or these) horizontal tangent(s). Round to the nearest hundredth.

APPLICATIONS

56. **Bacterial growth:** It is estimated that the population of a bacterial culture after t hours is approximately $N(t)=\frac{t^2-2t}{3\sqrt{t}+2}$, where $N(t)$ is in thousands and $2 \le t \le 10$. Find the rate of growth after 4 hours.

57. **Marginal revenue:** The demand function for a particular item is given by $D(x)=\frac{115}{3x+1}$. Find the marginal revenue when $x = 3$.

58. **Marginal profit:** The profit from the sale of x items is given by $P(x)=(2-0.5x)(0.5x-5)$, where $P(x)$ is in hundreds of dollars and $2 \le x \le 10$. Find the marginal profit when $x = 5$.

59. **Marginal cost:** The cost of producing x items of a product is given by $C(x)=(0.1x+100)(0.1x+20)-600$. Find the marginal cost when $x = 60$.

60. **Velocity of a particle:** A particle is moving slowly along a line. Its position after t seconds is $S(t)=\frac{t}{t^2+4}$ feet. Find the velocity when the particle has been moving for 3 seconds.

61. **Population growth:** It is estimated that t years from now the population of a city will be $P(t)=(0.6t-7)(0.5t+6)+85$ in thousands. How fast will the population be growing in 10 years?