

### Example 5: Marginal Propensity

If the marginal propensity to save is  $S'(x) = 0.02x$ , what is the marginal propensity to consume?

### Solution

$$\begin{aligned} C'(x) &= 1 - S'(x) \\ &= 1 - 0.02x \end{aligned}$$

We replace  $S'(x)$  with  $0.02x$ .

## 2.9 EXERCISES

### PRACTICE

In Exercises 1–10, create an appropriate function of the type indicated.

- The cost of producing  $x$  leather belts is given by  $C(x) = 220 + 0.4x$ . Determine the average cost function.
- The cost of manufacturing a certain class of screws for wall hangers is given by  $C(x) = 800 + 0.0005x + 0.00002x^2$ . Determine the average cost function.
- A vendor charges \$3 for a hot dog. What is the demand function? What is the revenue function?
- The Knoll Industrial Supply Company charges for 55-gallon drums using the demand function  $p(x) = 22.5 - 0.5x$ . What is the revenue function?
- A supplier of souvenir T-shirts charges street vendors for each order based on a setup fee of \$50 and an item charge of \$1.15 per T-shirt. What is the cost function for the vendor?
- A supplier of souvenir T-shirts has setup costs of \$25 and shirts cost \$0.40 each. What is the cost function?
- The answer to Exercise 5 is also a revenue function for the supplier of T-shirts. Using the answers to Exercises 5 and 6, determine a profit function for the supplier of T-shirts.
- For the supplier in Exercise 6, what is the average cost function?
- A department store's cost estimate for a line of rocker-recliner chairs is given by  $C(x) = 100 + 150x$ . The store sells them for \$450 each.
  - What is the average cost function?
  - What is the demand function?
  - What is the revenue function?
  - What is the profit function?

10. A shoe store estimates a certain line of dress shoes has costs given by  $C(x) = 1500 + 30x + 0.05x^2$ . The store charges \$75 per pair.
- What is the average cost function?
  - What is the demand function?
  - What is the revenue function?
  - What is the profit function?
11. Suppose that total national consumption is given by a function  $C(x) = 200 - 0.6x - 0.05x^{0.6}$ , where  $x$  is the total national income.
- Determine the marginal propensity to consume.
  - Determine the marginal propensity to save.
12. If the marginal propensity to save of a certain country is given by  $S'(x) = 0.4x + 0.3$ , determine the marginal propensity to consume.
13. The average cost  $\bar{C}(x)$  of a product is  $\frac{C(x)}{x}$ , where  $C(x)$  is the total cost function.
- What is the average cost function if its total cost function is  $C(x) = 30 + 2x + 0.003x^2$ ?
  - What is the rate of change of average cost?
  - What value of  $x$  results in a minimum average cost?
14. The average cost of a product is given by  $A(x) = 20x^{-1} + 3$ .
- Determine the cost function for the product.
  - Determine the marginal cost function.

### APPLICATIONS

15. The weekly cost of producing  $x$  electric drills is given by the function

$$C(x) = 2400 + 28x + 0.25x^2.$$

- Find  $C(10)$ ,  $C(20)$ , and  $C(30)$ .
  - Find the marginal cost function.
  - Find  $C'(10)$ ,  $C'(20)$ , and  $C'(30)$ .
  - Find the average cost function and the marginal average cost function.
  - Find the marginal average cost when  $x = 10$ ,  $x = 20$ , and  $x = 30$ .
16. The total cost function for producing  $x$  units of a product is given by

$$C(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 7x + 18.$$

- Find  $C(3)$ ,  $C(4)$ , and  $C(6)$ .
- Find the marginal cost function.
- Find  $C'(3)$ ,  $C'(4)$ , and  $C'(6)$ .
- Find the average cost function and the marginal average cost function.
- Find the marginal average cost when  $x = 3$ ,  $x = 4$ , and  $x = 6$ .

17. The total cost of producing  $x$  units of a commodity is given by

$$C(x) = 60 + 10x - 0.5x^2.$$

- Find  $C(4)$ ,  $C(6)$ , and  $C(9)$ .
- Find the marginal cost function.
- Find  $C'(4)$ ,  $C'(6)$ , and  $C'(9)$ .
- Find the average cost function.
- Find the marginal average cost when  $x = 4$ ,  $x = 6$ , and  $x = 9$ .

18. The total cost of producing  $x$  wireless speakers is given by the function  $C(x) = 300 + 24x - 0.4x^2 + 0.1x^3$ .
- Find  $C(2)$ ,  $C(3)$ , and  $C(5)$ .
  - Find the marginal cost function.
  - Find  $C'(2)$ ,  $C'(3)$ , and  $C'(5)$ .
  - Find the average cost function and the marginal average cost function.
  - Find the marginal average cost if  $x = 4$ .
19. A manufacturer has determined that the revenue from the sale of  $x$  cell phones is given by  $R(x) = 94x - 0.03x^2$  dollars. The cost of producing  $x$  cell phones is  $C(x) = 10,800 + 34x$  dollars.
- Find the profit function  $P(x)$ .
  - Find  $P(200)$ ,  $P(400)$ , and  $P(600)$ .
  - Find the marginal profit function  $P'(x)$ .
  - Find  $P'(200)$ ,  $P'(400)$ , and  $P'(600)$ .
  - Find any break-even points.
20. The revenue from the sale of  $x$  fire extinguishers is estimated to be  $R(x) = 54x - 0.4x^2$  dollars. The total cost of producing  $x$  fire extinguishers is  $C(x) = 400 + 30x - 0.2x^2$  dollars.
- Find the profit function  $P(x)$ .
  - Find  $P(20)$ ,  $P(40)$ , and  $P(60)$ .
  - Find the marginal profit function  $P'(x)$ .
  - Find  $P'(20)$ ,  $P'(40)$ , and  $P'(60)$ .
  - Find any break-even points.
21. A company that produces and sells compact refrigerators has found that the revenue from the sale of  $x$  refrigerators is  $R(x) = 100x - 0.1x^2$  dollars. The cost function is given by  $C(x) = 2070 + 25x + 0.1x^2$  dollars.
- Find the profit function  $P(x)$ .
  - Find  $P(60)$ ,  $P(80)$ , and  $P(100)$ .
  - Find the marginal profit function  $P'(x)$ .
  - Find  $P'(60)$ ,  $P'(80)$ , and  $P'(100)$ .
  - Find any break-even points.
22. A manufacturer has determined that the cost and the revenue of producing and selling  $x$  telescopes are  $C(x) = x^2 + 20x + 1050$  dollars and  $R(x) = 140x - 0.5x^2$  dollars, respectively.
- Find the profit function  $P(x)$ .
  - Find  $P(30)$ ,  $P(35)$ , and  $P(40)$ .
  - Find the marginal profit function  $P'(x)$ .
  - Find  $P'(30)$ ,  $P'(35)$ , and  $P'(40)$ .
  - Find any break-even points.

23. The owner of a leather craft shop has determined that he can sell  $x$  attaché cases if the price is  $p = D(x) = 46 + 0.25x$  dollars. The total cost for these cases is  $C(x) = 0.15x^2 + 6x + 190$  dollars.
- Find the revenue function  $R(x)$ .
  - Find the profit function  $P(x)$ .
  - Find  $P(25)$ ,  $P(30)$ , and  $P(40)$ .
  - Find the marginal profit function  $P'(x)$ .
  - Find  $P'(25)$ ,  $P'(30)$ , and  $P'(40)$ .
24. A firm can sell  $x$  items of a product when the price is  $p = D(x) = 3.00 - 0.001x$  dollars. The total production costs are  $C(x) = 0.002x^2 + 0.72x + 260$  dollars.
- Find the revenue function  $R(x)$ .
  - Find the profit function  $P(x)$ .
  - Find  $P(300)$ ,  $P(375)$ , and  $P(400)$ .
  - Find the marginal profit function  $P'(x)$ .
  - Find  $P'(300)$ ,  $P'(375)$ , and  $P'(400)$ .
25. A local publishing company prints a special magazine each month. It has been determined that  $x$  magazines can be sold monthly when the price is  $p = D(x) = 5.50 - 0.0004x$ . The total cost of producing the magazine is  $C(x) = 0.0002x^2 + x + 4650$  dollars.
- Find the revenue function  $R(x)$ .
  - Find the profit function  $P(x)$ .
  - Find  $P(3000)$ ,  $P(3500)$ , and  $P(4000)$ .
  - Find the marginal profit function  $P'(x)$ .
  - Find  $P'(3000)$ ,  $P'(3500)$ , and  $P'(4000)$ .
26. A sales representative for a company that produces skateboards can sell  $x$  units of their deluxe model if the price is  $p = D(x) = 79.9 - 0.03x$  dollars. The total cost for these skateboards is given by  $C(x) = 0.08x^2 + 5.1x + 5800$  dollars.
- Find the revenue function  $R(x)$ .
  - Find the profit function  $P(x)$ .
  - Find  $P(320)$ ,  $P(340)$ , and  $P(350)$ .
  - Find the marginal profit function  $P'(x)$ .
  - Find  $P'(320)$ ,  $P'(340)$ , and  $P'(350)$ .
27. A certain model car has a valuation in dollars given by the formula  $f(x) = 12,519.3 - 1391.1x$ , for  $0 \leq x \leq 7$ , where  $x$  is the age of the car in years.  $x = 0$  corresponds to this calendar year.
- What is  $f(0)$ ? Interpret this number.
  - What is the marginal valuation? Interpret this number.
28. Based on averaging results at a certain state college, a relationship between grades and SAT scores was found to be  $f(s) = 1.36 + 0.00141s$ , where  $s$  is a student's SAT score and  $f(s)$  is the student's graduating GPA (GPA based on 4.0 maximum score).
- What is the expected GPA for a student with an SAT score of 1000?
  - What is the marginal GPA? Interpret this number.

Exercises 29–31 deal with projections of world population based on estimates of fertility around the world. Use the following background information:

The Total Fertility Rate (TFR) is the average number of children a woman will have. The United Nations projects world population according to assumptions about TFR values. In general, lower values promote economic well-being and lower world population. (Source: Population Reference Bureau, “Transitions in World Population,” *Population Bulletin*, Vol. 59, No. 1, 36, March 2004.)

**29. Total fertility rate:** Using a TFR of 1.5, the UN projects total world population, in billions, will be modeled by  $F(t)$  where  $t$  is the number of years after 2000 and

$$F \text{ is the function } F(t) = -\frac{2.1}{10^6}t^4 + \frac{1.6}{10^4}t^3 - 0.00457t^2 + 0.0994t + 6.03.$$

- Give the marginal population function  $F'(t)$ .
- Determine the estimate of world population in 2030.
- What is the marginal population in 2030? Interpret this number.

**30. Total fertility rate:** Using a TFR of 2.0, the UN projects total world population, in billions, will be modeled by  $G(t)$  where  $t$  is the number of years after 2000 and

$$G \text{ is the function } G(t) = -\frac{2.1}{10^6}t^4 + \frac{1.66}{10^4}t^3 - 0.0045t^2 + 0.113t + 6.03.$$

- Give the marginal population function  $G'(t)$ .
- Determine the estimate of world population in 2030.
- What is the marginal population in 2030? Interpret this number.

**31. Total fertility rate:** Using a TFR of 2.5, the UN projects total world population, in billions, will be modeled by  $H(t)$  where  $t$  is the number of years after 2000 and

$$H \text{ is the function } H(t) = -\frac{2.6}{10^6}t^4 + \frac{1.87}{10^4}t^3 - 0.00396t^2 + 0.116t + 6.05.$$

- Give the marginal population function  $H'(t)$ .
- Determine the estimate of world population in 2030.
- What is the marginal population in 2030? Interpret this number.



### WRITING & THINKING

**32.** An economics professor claimed that the average cost was a minimum if the average cost equaled the marginal cost. Do you agree? Explain why or why not.

**33.** “The average revenue  $\frac{R(x)}{x}$  is not usually studied in the context of business economics.” Argue for or against this statement.