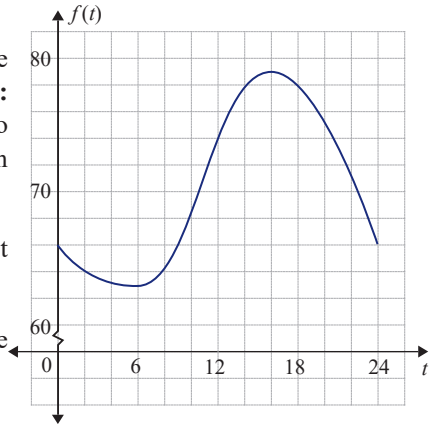
2.6 EXERCISES

PRACTICE

Use the graph to solve Exercises 1–3. The variable t is the number of hours since midnight and $f(t)$ is the temperature at time t.



1. Roughly estimate the instantaneous rate of change of $f(t)$ at 3:00 p.m. (**Hint:** Extend an imaginary tangent line so as to come close to or to intersect points with integer coordinates.)
2. Estimate the time t for the lowest temperature $f(t)$.
3. Estimate the time t for the fastest increase in temperature $f(t)$.
4. Sketch one graph so that all of the following statements are true.

   (a) $f'(x)$ is positive for $-2 \le x \le 6$.

   (b) $f'(x) < 0$ for $x > 6$

   (c) $f'(6) = 0$

   (d) $f(6) = 10$ and $f(0) = 1$
5. Interpret the meaning of $f(3) = 14$ and $f'(3) = 7$ for the function $f(x) = 2 + x + x^2$.

APPLICATIONS

In Exercises 6–15, determine what the slope f' represents in terms of the subject in the problem.

6. $f(t)$ is the distance in feet traveled by a car in t minutes.
7. $f(s)$ is the total money spent in a department store by s customers.
8. $f(n)$ is the number of birds nesting in woods with n trees per acre.
9. $f(x)$ is the total cost of manufacturing x toasters.
10. $f(u)$ is the total revenue from the sale of u car radios.
11. $f(t)$ is the speed of a race car after t seconds.
12. $f(v)$ is the total amount of information in bytes fed into a server at Castle Manufacturing Company in v seconds.
13. $f(x)$ is the vertical distance in meters traveled by a test rocket in x seconds.
14. $f(s)$ is the grade point average of freshmen at Sullivan Technical College where s is the average SAT score of the freshman class.
15. $f(t)$ is the cost of calculus books at college bookstores in the United States where t is the time in years since 1980.

16. Suppose $f(x)$ is the number of gallons of gas used by a car after it has traveled x miles.
- **a.** Suppose the car gets 20 miles/gallon. What is $f(100)$?
- **b.** Is $f'(100)$ positive or negative?
- **c.** Would $f'(100)$ be greater for a subcompact car or for an SUV?

17. Suppose $f(x)$ denotes the production units for input x in labor units (man-hours). Suppose $f(500) = 2000$ and $f'(500) = 3$.
- **a.** Interpret $f(500) = 2000$ and $f'(500) = 3$.
- **b.** Estimate the increased production if x is increased from 500 to 501.

18. Suppose $f(x)$ is the total number of students on a college campus that have the flu and x is the number of days after the first case is reported. Interpret $f(8) = 9$ and $f'(8) = 3$.

19. Suppose $f(x)$ is the cost of a Toyota Camry and x is the age of the car.
- **a.** Is $f'(x)$ positive or negative?
- **b.** Interpret the meaning of $f'(3) = -2500$.

20. Average prices for one-bedroom condominiums have steadily risen in Charleston, SC since 2000, according to local reports. Suppose $f(x) = 3000x + 72{,}000$ is the cost of a one-bedroom condo and x is the number of years since 2000.
- **a.** Interpret $f(0) = 72{,}000$.
- **b.** Interpret $f'(x) = 3000$.
- **c.** Interpret $f(3) = 81{,}000$.

21. Suppose $f(x)$ denotes the weight of a cancerous tumor x weeks after discovery. Interpret $f(3) = 4$ grams and $f'(3) = 0.4$ grams/week.

22. Water boils at 212 °F and at 100 °C. Water freezes at 32 °F and 0 °C. Let F denote temperature in degrees Fahrenheit and let x be temperature in degrees Celsius.
- **a.** Write a formula $F(x) = mx + b$, which can convert Celsius input x into Fahrenheit output F.
- **b.** Use the value of m to write a formula for $F'(x)$.

23. Birth rate: The fertility decline in many countries can be modeled by an appropriate equation. In Bangladesh, from 1970 to 2000, patterns of fertility changed according to the equation $y = -0.11x + 6.45$, where x is the time in years beginning in 1970 and y is the average number of children per woman. (**Source:** Lori Ashford, "World Population Highlights 2004," BRIDGE Population Reference Bureau, August 2004.)
- **a.** What number is $f(20)$ and what does it represent?
- **b.** What number is $f'(20)$ and what does it represent?

24. Birth rate: Patterns of fertility changed in India from 1970 to 2000 according to the equation $y = -0.068x + 5.22$, where x is the number of years after 1970 and y is the average number of children per woman. (**Source:** Lori Ashford, "World Population Highlights 2004," BRIDGE Population Reference Bureau, August 2004.)
- **a.** What number is $f(30)$ and what does it represent?
- **b.** What number is $f'(30)$ and what does it represent?

25. Birth rate: In China, from 1964 to the present, the death rate has remained nearly constant at about 8 deaths per 1000 persons. However, the yearly birth rate, in births per 1000 people, has declined in most years according to the formula $f(x) = -0.641x + 35.8$, where x is the number of years since 1964. (**Source:** Nancy E. Riley, "China's Population: New Trends and Challenges," *Population Bulletin*, Vol. 20, No. 2, June 2004.)

a. What number is $f(30)$ and what does it represent?

b. What number is $f'(30)$ and what does it represent?

(**Note:** See Exercise 36 in Section 2.8 for a similar problem using a more accurate model than the linear model given here. Both models are based on the same data.)

TECHNOLOGY

Use a graphing calculator in Exercises 26–29 to find the slope of $f(x)$ at the given point. Sketch the graph of $f(x)$ and the tangent line at the given point on your paper.

26. $f(x) = \dfrac{4+2x}{\sqrt{x}}; (16, 9)$

27. $f(x) = x^3; (3, 27)$

28. $f(x) = 2 - 3x + x^2; (1, 0)$

29. $f(x) = 10^x; (2, 100)$

30. For the function $f(x) = 4 - x - 2x^2$, find a window including the point (0, 4) so that the graph of the function and the tangent at (0, 4) are indistinguishable.

31. For the function $f(x) = x^3$, make a table with headings a, $f(a)$, and $f'(a)$. Then substitute numbers using $a = -1, 0, 1, 2, 3, 4$. Give a formula for $f'(x)$.

32. Locate (with a graphing calculator) the x- and y-coordinates of the lowest point on the graph $f(x) = x^2 - 6x + 11$. What is the slope at the lowest point?

33. For the function $y = x^2$ add a column to Table 3 to include the y-intercepts of the tangent line. What curiosity do you observe in the table?

34. Find the ln button on your graphing calculator and sketch a graph of $y = \ln x$ on your calculator using the window $[-2, 8]$ by $[-0.5, 2]$. What is the slope at the point (1, 0)? Sketch the graph and tangent on your paper.

35. On $f(x) = \sqrt{x}$, locate the x- and y-coordinates of the point at which the slope is exactly 1. (**Hint:** Find $\left(a, \sqrt{a}\right)$ so that $f'(a) = 1$.)

36. A bacteria culture in a lab grows according to the formula $y = 1600\left(2^t\right)$ where t is time in hours and y is the quantity of bacteria.

a. Interpret the meaning of $f'(1)$.

b. Determine $f'(1)$ using a graphing calculator.

37. Sketch $f(x) = (x+10)(x-5)(x-10)$ on a graphing calculator. Give the window used.

a. Locate the points $(a, f(a))$ for which $f'(a) = 0$.

b. What is the value of $f'(2)$?

38. Sketch $y = \frac{1}{x}$ on a graphing calculator. Find the x- and y-coordinates of any point with slope -25.

39. Suppose $f(x) = 2x^2$. Create a table of values for the slope of $f'(x)$ (see Table 4). Guess a formula for $f'(x)$.