Indeterminate Form

A limit expression of the type $\lim_{x \to a} \left(\frac{g(x)}{f(x)} \right)$ is called an **indeterminate form** of type $\frac{0}{0}$ if $\lim_{x \to a} g(x) = 0$ and $\lim_{x \to a} f(x) = 0$.

A strategy of solving such a problem is given by the two-step method illustrated in the previous example.

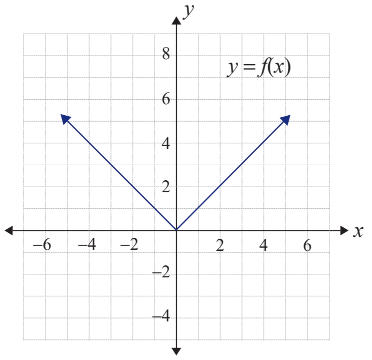
1. Replace the quotient with a simplified expression after factoring.
2. Evaluate the new limit problem by substitution if the denominator does not have a limit of 0 as $x \to a$.

2.2 EXERCISES

PRACTICE

In Exercises 1–12, use the graph to find the indicated limits, if they exist.

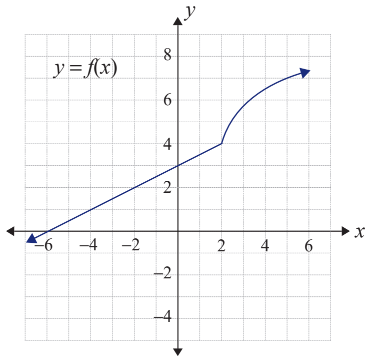
1.



a. $\lim_{x \to 0^-} f(x)$ **b.** $\lim_{x \to 0^+} f(x)$

c. $\lim_{x \to 0} f(x)$ **d.** $\lim_{x \to 2} f(x)$

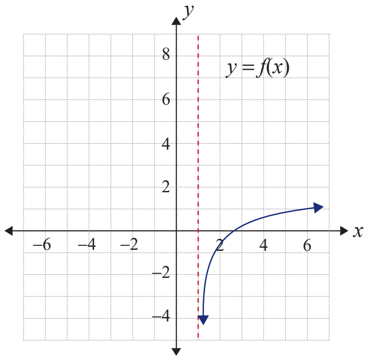
2.



a. $\lim_{x \to 2^-} f(x)$ **b.** $\lim_{x \to 2^+} f(x)$

c. $\lim_{x \to 2} f(x)$ **d.** $\lim_{x \to 0} f(x)$

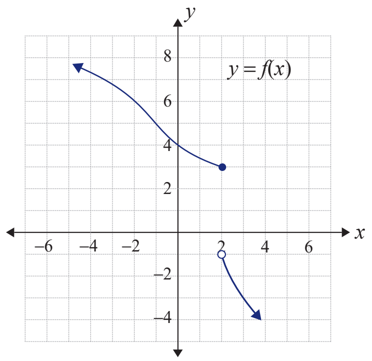
3.



a. $\lim_{x \to 1^+} f(x)$ **b.** $\lim_{x \to 6^-} f(x)$

c. $\lim_{x \to 6^+} f(x)$ **d.** $\lim_{x \to 6} f(x)$

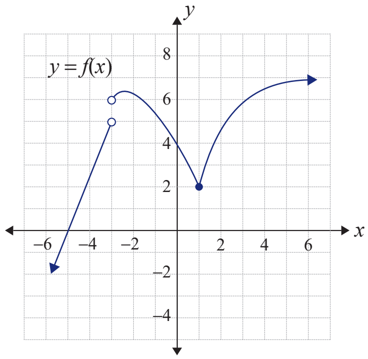
4.



a. $\lim_{x \to 2^-} f(x)$ **b.** $\lim_{x \to 2^+} f(x)$

c. $\lim_{x \to 2} f(x)$ **d.** $\lim_{x \to 0} f(x)$

5.

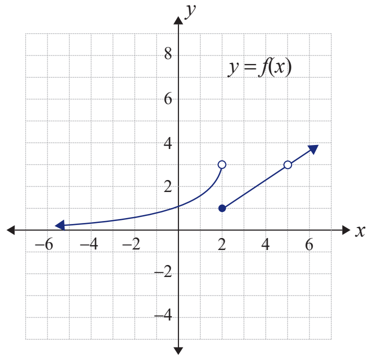


a. $\lim_{x\to -3^-} f(x)$ **b.** $\lim_{x\to -3^+} f(x)$

c. $\lim_{x\to -3} f(x)$ **d.** $\lim_{x\to 1^-} f(x)$

e. $\lim_{x\to 1^+} f(x)$ **f.** $\lim_{x\to 1} f(x)$

6.

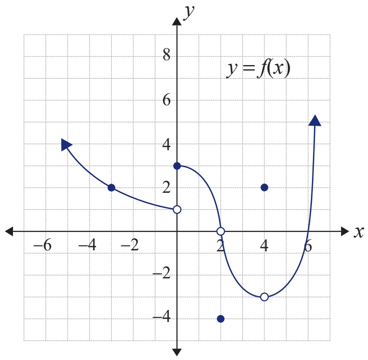


a. $\lim_{x\to 2^-} f(x)$ **b.** $\lim_{x\to 2^+} f(x)$

c. $\lim_{x\to 2} f(x)$ **d.** $\lim_{x\to 5^+} f(x)$

e. $\lim_{x\to 5^-} f(x)$ **f.** $\lim_{x\to 5} f(x)$

7.

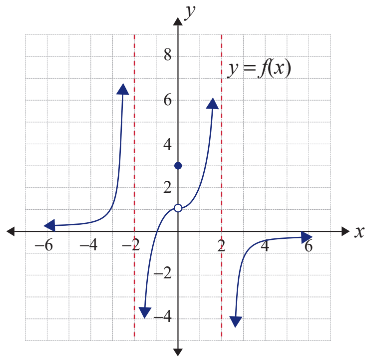


a. $\lim_{x\to 0^-} f(x)$ **b.** $\lim_{x\to 0^+} f(x)$

c. $\lim_{x\to 0} f(x)$ **d.** $\lim_{x\to 4^-} f(x)$

e. $\lim_{x\to 4^+} f(x)$ **f.** $\lim_{x\to 4} f(x)$

8.

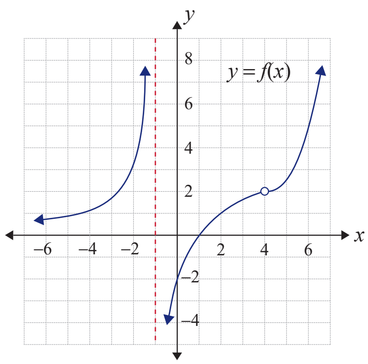


a. $\lim_{x\to -2^-} f(x)$ **b.** $\lim_{x\to -2^+} f(x)$

c. $\lim_{x\to 0^-} f(x)$ **d.** $\lim_{x\to 0^+} f(x)$

e. $\lim_{x\to 0} f(x)$ **f.** $\lim_{x\to 3} f(x)$

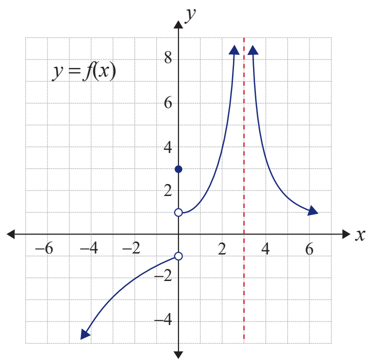
9.



a. $\lim_{x\to -1^-} f(x)$ **b.** $\lim_{x\to -1^+} f(x)$

c. $\lim_{x\to 4} f(x)$ **d.** $\lim_{x\to 1} f(x)$

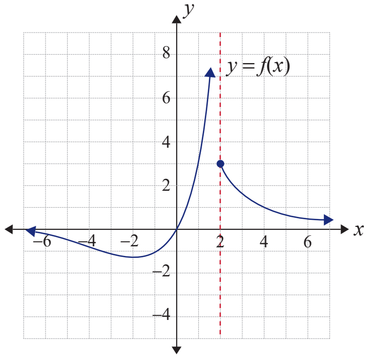
10.



a. $\lim_{x\to 0^-} f(x)$ **b.** $\lim_{x\to 0^+} f(x)$

c. $\lim_{x\to 0} f(x)$ **d.** $\lim_{x\to 3^-} f(x)$

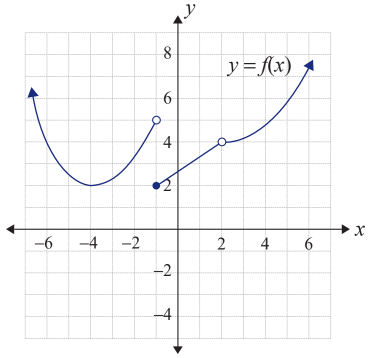
11.



a. $\lim_{x\to 2^-} f(x)$ **b.** $\lim_{x\to 2^+} f(x)$

c. $\lim_{x\to 2} f(x)$ **d.** $\lim_{x\to 0} f(x)$

12.



a. $\lim_{x\to -1^-} f(x)$ **b.** $\lim_{x\to -1^+} f(x)$

c. $\lim_{x\to -1} f(x)$ **d.** $\lim_{x\to 2} f(x)$

In Exercises 13–19, determine the limit by first simplifying the expression algebraically.

13. $\lim_{x\to 0}\left(\dfrac{x^3+2x}{2x^2+x}\right)$

14. $\lim_{x\to 3}\left(\dfrac{3-13x+4x^2}{x-3}\right)$

15. $\lim_{x\to 6}\left(\dfrac{x^2-36}{x-6}\right)$

16. $\lim_{x\to -7}\left(\dfrac{x-7}{x^2-49}\right)$

17. $\lim_{h\to 0}\left(\dfrac{f(3+h)-f(3)}{h}\right), f(x)=x^2-2$

18. $\lim_{h\to 0}\left(\dfrac{f(2-h)-f(2)}{h}\right), f(x)=1-x+x^2$

19. $\lim_{x\to 4}\left(\dfrac{x^4-256}{x^2-16}\right)$

APPLICATIONS

20. Salary: Erin is paid a weekly salary of $12 per hour plus time-and-a-half for overtime (time in excess of 40 hours, but no more than 60 hours). Her salary is given by the function

$$S(t)=\begin{cases}12t & \text{if } 0<t\le 40\\ 480+18(t-40) & \text{if } 40<t\le 60\end{cases}$$

where t is the time in hours, $0<t\le 60$.

a. Find $\lim_{t\to 40^-} S(t)$. **b.** Find $\lim_{t\to 40^+} S(t)$. **c.** Find $\lim_{t\to 40} S(t)$.

WRITING & THINKING

21. Suppose $f(x)$ and $g(x)$ are equal for all x-values except $x=t$.

a. Is $\lim_{x\to t^-} f(x)=\lim_{x\to t^-} g(x)$ true?

b. What about $\lim_{x\to t^+} f(x)=\lim_{x\to t^+} g(x)$?

c. Is $\lim_{x\to t} f(x)=\lim_{x\to t} g(x)$ necessarily true?