2.1 EXERCISES

PRACTICE

In Exercises 1–6, determine the limits. In each case, make a suitable table, with four values, to support your answer. Choose the fourth value ±0.001 from the indicated *a*-value.

1. $\lim\limits_{x\to 7^-}\left(\dfrac{x^2-49}{x-7}\right)$

2. $\lim\limits_{x\to 7^+}\left(\dfrac{x^2+49}{x-7}\right)$

3. $\lim\limits_{x\to 3^+}\left(\dfrac{x^3-9x^2+27x-27}{x-3}\right)$

4. $\lim\limits_{h\to 0^+}\left(\dfrac{\sqrt{4+h}}{h}\right)$

5. $\lim\limits_{a\to 1^+}\left(\dfrac{a^{10}-1}{a-1}\right)$

6. $\lim\limits_{n\to \sqrt{2}^-}\left(\dfrac{n^2-2}{n-\sqrt{2}}\right)$

Given the table for $\lim\limits_{x\to a^+} f(x)$ in Exercise 7 and $\lim\limits_{x\to a^-} f(x)$ in Exercises 8–9, **a.** give the value for *a* and **b.** determine the limit, if there is one.

7.

x	y
2.500	0.2222
2.100	0.2439
2.010	0.2494
2.001	0.2499

8.

x	y
3.800	15.60
3.900	15.80
3.990	15.98
3.999	15.998

9.

x	y
3.000	3.43
3.100	11.99
3.140	313.90
3.141	843.60

10. a. Complete the table.

x	$f(x)=3x-1$
1	
1.4	
1.8	
1.9	
1.99	
1.999	

b. Find $\lim\limits_{x\to 2^-}(3x-1)$.

11. a. Complete the table.

x	$f(x)=x^2-2$
0	
−0.4	
−0.8	
−0.9	
−0.99	
−0.999	

b. Find $\lim\limits_{x\to -1^+}(x^2-2)$.

12. a. Complete the table.

x	$f(x)=\dfrac{x^2-1}{x+1}$
2	
1.6	
1.2	
1.1	
1.01	
1.001	

b. Find $\lim\limits_{x\to 1^+}\left(\dfrac{x^2-1}{x+1}\right)$.

13. a. Complete the table.

x	$f(x)=x^2+3$
2	
2.4	
2.8	
2.9	
2.99	
2.999	

b. Find $\lim\limits_{x\to 3^-}(x^2+3)$.

14. a. Complete the table.

x	$f(x)=\dfrac{1}{x-4}$
3	
3.4	
3.8	
3.9	
3.99	
3.999	

b. Find $\lim\limits_{x\to 4^-}\left(\dfrac{1}{x-4}\right)$.

15. a. Complete the table.

x	$f(x)=\dfrac{x}{x+2}$
−3	
−2.6	
−2.2	
−2.1	
−2.01	
−2.001	

b. Find $\lim\limits_{x\to -2^-}\left(\dfrac{x}{x+2}\right)$.

16. a. Complete the table.

x	$f(x)=\dfrac{x^2-4}{x+2}$
−1	
−1.4	
−1.8	
−1.9	
−1.99	
−1.999	

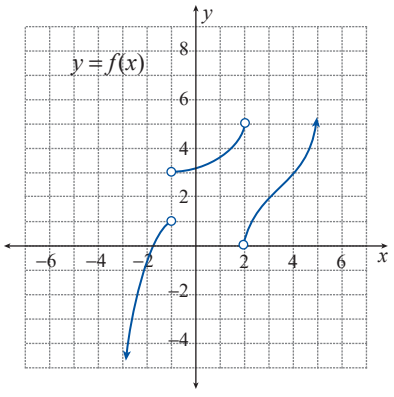
b. Find $\lim\limits_{x\to -2^+}\left(\dfrac{x^2-4}{x+2}\right)$.

17. a. Complete the table.

x	$f(x)=\dfrac{x-3}{x^2-2x-3}$
4	
3.6	
3.2	
3.1	
3.01	
3.001	

b. Find $\lim\limits_{x\to 3^+}\left(\dfrac{x-3}{x^2-2x-3}\right)$.

Find In Exercises 18–23, use the graph of $y=f(x)$ to find the limits.



18. $\lim\limits_{x\to -1^-} f(x)$

19. $\lim\limits_{x\to -1^+} f(x)$

20. $\lim\limits_{x\to 2^-} f(x)$

21. $\lim\limits_{x\to 2^+} f(x)$

22. $\lim\limits_{x\to 3^-} f(x)$

23. $\lim\limits_{x\to 3^+} f(x)$

In Exercises 24–29, use the graph of $y = f(x)$ to find the limits.

24. $\lim\limits_{x \to -1^-} f(x)$

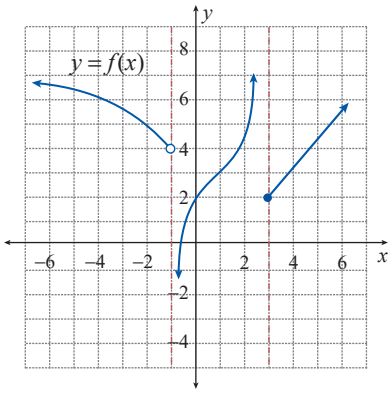
25. $\lim\limits_{x \to -1^+} f(x)$

26. $\lim\limits_{x \to 0^-} f(x)$

27. $\lim\limits_{x \to 0^+} f(x)$

28. $\lim\limits_{x \to 3^-} f(x)$

29. $\lim\limits_{x \to 3^+} f(x)$



In Exercises 30–35, use the graph of $y = f(x)$ to find the limits.

30. $\lim\limits_{x \to 0^-} f(x)$

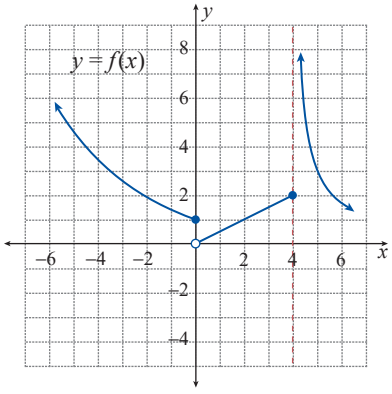
31. $\lim\limits_{x \to 0^+} f(x)$

32. $\lim\limits_{x \to 4^-} f(x)$

33. $\lim\limits_{x \to 4^+} f(x)$

34. $\lim\limits_{x \to 2^-} f(x)$

35. $\lim\limits_{x \to 2^+} f(x)$



Find the one-sided limits indicated in Exercises 36–59.

36. $\lim\limits_{x \to 2^+} (5x - 3)$

37. $\lim\limits_{x \to -1^+} (2x + 7)$

38. $\lim\limits_{x \to 0^-} (4 - 3x)$

39. $\lim\limits_{x \to 3^-} (1 - 6x)$

40. $\lim\limits_{x \to 2^-} (x^2 - 3x + 1)$

41. $\lim\limits_{x \to -5^+} (x^2 + 4x - 2)$

42. $\lim\limits_{x \to -4^+} (x^2 - x + 3)$

43. $\lim\limits_{x \to -3^-} (x^2 + 2x - 3)$

44. $\lim\limits_{x \to 10^-} (0.01x^2 + 7x - 30)$

45. $\lim\limits_{x \to 10^+} (0.2x^2 - 5x + 6)$

46. $\lim\limits_{x \to 0^+} \left(\dfrac{x-3}{x} \right)$

47. $\lim\limits_{x \to 0^-} \left(\dfrac{2x+1}{x} \right)$

48. $\lim\limits_{x \to 1^+} \left(\dfrac{x-2}{x-1} \right)$

49. $\lim\limits_{x \to 1^-} \left(\dfrac{x-2}{x-1} \right)$

50. $\lim\limits_{x \to 2^-} \left(\dfrac{1}{x+2} \right)$

51. $\lim\limits_{x \to 2^+} \left(\dfrac{1}{x+2} \right)$

52. $\lim\limits_{x \to 3^+} \left(\dfrac{1}{x+1} \right)$

53. $\lim\limits_{x \to 1^+} \left(\dfrac{1}{x-5} \right)$

54. $f(x)=\begin{cases}2-3x & \text{if } x<2\\ x-1 & \text{if } x\geq 2\end{cases}$

a. $\lim\limits_{x\to 2^-} f(x)$

b. $\lim\limits_{x\to 2^+} f(x)$

55. $f(x)=\begin{cases}x^2+2 & \text{if } 0\leq x\leq 3\\ 2x+5 & \text{if } x>3\end{cases}$

a. $\lim\limits_{x\to 3^-} f(x)$

b. $\lim\limits_{x\to 3^+} f(x)$

56. $f(x)=\begin{cases}3x+1 & \text{if } 0\leq x\leq 4\\ x^2-3 & \text{if } x>4\end{cases}$

a. $\lim\limits_{x\to 4^-} f(x)$

b. $\lim\limits_{x\to 4^+} f(x)$

57. $f(x)=\begin{cases}x^3 & \text{if } x<2\\ x^2+5 & \text{if } x\geq 2\end{cases}$

a. $\lim\limits_{x\to 2^-} f(x)$

b. $\lim\limits_{x\to 2^+} f(x)$

58. $f(x)=\begin{cases}3-2x & \text{if } x<1\\ x & \text{if } 1\leq x\leq 4\\ \dfrac{1}{x-4} & \text{if } x>4\end{cases}$

a. $\lim\limits_{x\to 1^-} f(x)$ **b.** $\lim\limits_{x\to 1^+} f(x)$

c. $\lim\limits_{x\to 4^-} f(x)$ **d.** $\lim\limits_{x\to 4^+} f(x)$

59. $f(x)=\begin{cases}x^2-1 & \text{if } 0\leq x<2\\ 3 & \text{if } 2\leq x\leq 5\\ \dfrac{1}{x-5} & \text{if } x>5\end{cases}$

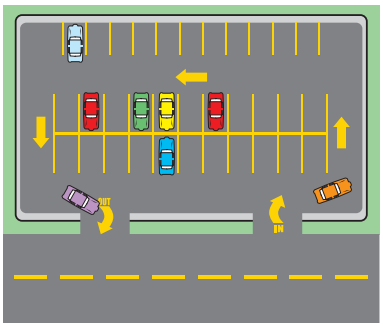
a. $\lim\limits_{x\to 2^-} f(x)$ **b.** $\lim\limits_{x\to 2^+} f(x)$

c. $\lim\limits_{x\to 5^-} f(x)$ **d.** $\lim\limits_{x\to 5^+} f(x)$

APPLICATIONS

60. Parking rates: The rate for parking in the short-term lot (maximum of 24 hours) at the airport is \$1.00 for the first hour plus \$0.75 for each additional hour or part thereof, with a maximum cost of \$7.00. The function for the cost of parking on this lot for t hours (up to 24 hours) is as follows.

$$C(t)=\begin{cases}1.00 & \text{for } 0<t\leq 1\\ 1.75 & \text{for } 1<t\leq 2\\ 2.50 & \text{for } 2<t\leq 3\\ 3.25 & \text{for } 3<t\leq 4\\ 4.00 & \text{for } 4<t\leq 5\\ 4.75 & \text{for } 5<t\leq 6\\ 5.50 & \text{for } 6<t\leq 7\\ 6.25 & \text{for } 7<t\leq 8\\ 7.00 & \text{for } 8<t\leq 24\end{cases}$$



a. Graph the function for $0<t\leq 24$ hr.

b. Find $\lim\limits_{t\to 3^-} C(t)$.

c. Find $\lim\limits_{t\to 3^+} C(t)$.

d. Find $\lim\limits_{t\to 8^-} C(t)$.

e. Find $\lim\limits_{t\to 8^+} C(t)$.

WRITING & THINKING

61. Suppose $f(x)$ and $g(x)$ are polynomials and $f(t)=0=g(t)=0$ for some t. If $\lim\limits_{x\to t^-}\dfrac{f(x)}{g(x)}=L$, must $\lim\limits_{x\to t^+}\dfrac{f(x)}{g(x)}$ also be L?