10.2 EXERCISES

PRACTICE

1. Find the linear approximation to $y = e^x + \ln x$ near $x = 1$.

In Exercises 2–9, find the Taylor polynomial of degree k approximating the given function near $x = 0$. Using a graphing calculator, graph the given function and the Taylor approximation in the same window.

2. $y = \dfrac{1}{\sqrt{1+x}}, \quad k = 3$

3. $y = \cos x, \quad k = 6$

4. $y = \sqrt[3]{x+2}, \quad k = 2$

5. $y = \sqrt{x+4}, \quad k = 3$

6. $y = x\ln(x+1), \quad k = 3$

7. $y = \ln(x+1), \quad k = 3$

8. $y = xe^x, \quad k = 3$

9. $y = e^{x+1}, \quad k = 3$

In Exercises 10–17, find the Taylor polynomial of degree k near $x = a$.

10. $y = \sin x, \quad a = \pi, \quad k = 5$

11. $y = e^x, \quad a = 1, \quad k = 4$

12. $y = e^{3x}, \quad a = 1, \quad k = 4$

13. $y = \cos x, \quad a = \pi, \quad k = 6$

14. $y = x^{\frac{1}{3}}, \quad a = 1, \quad k = 4$

15. $y = \sqrt{x}, \quad a = 1, \quad k = 4$

16. $y = \dfrac{x}{1+x}, \quad a = 0, \quad k = 5$

17. $y = \dfrac{1}{1-x}, \quad a = 0.5, \quad k = 4$

18. Use the Taylor polynomial of degree 6 for $y = \cos x$, centered at $x = 0$, to estimate the area under the graph of $y = \cos x$ from $x = 0$ to $x = 1.5708$.

19. Use the Taylor polynomial of degree 5 for $y = \sin x$, centered at $x = 0$, to estimate the area under the graph of $y = \sin x$ from $x = 0$ to $x = 3.14159$.

20. Construct the Taylor polynomial of degree 3, centered at $x = 2$, for the function $f(x) = \sqrt{x+1}$. Use this polynomial to estimate a value for $\sqrt{2}$. Compare this to the value given by a calculator.

21. Use the Taylor polynomial of degree 3 for $f(x) = \sqrt{x+1}$, centered at $x = 0$, to estimate a value for $\sqrt{2}$. Compare this to the value given by a calculator.

22. Use the Taylor polynomial of degree 3, centered at $x = 1$, for $f(x) = \sqrt[4]{x}$ to estimate the value of $\sqrt[4]{0.5}$. Compare your value with the value given by a calculator.

23. Use the Taylor polynomial of degree 3, centered at $x = 0$, for $f(x) = \sqrt[4]{x+1}$ to estimate the value of $\sqrt[4]{0.5}$ (that is, calculate $P_3(-0.5)$). Compare this with your answer to Exercise 22.

24. Choose an appropriate function $y = f(x)$, a suitable value of $x = a$, and a corresponding Taylor polynomial of degree 4 in order to estimate $\sqrt{37}$.

25. Use a quadratic Taylor polynomial for an appropriate function $f(x)$ and an appropriate value of $x = a$ in order to estimate $\sqrt[3]{27.5}$.

26. **a.** Multiply the Taylor polynomial of degree 3 for $y = \sin x$ (centered at $x = 0$) by itself.
b. Is the result equal to the Taylor polynomial of degree 6 (centered at $x = 0$) for $y = \sin^2 x$?

27. Use the identity $\cos^2 x + \sin^2 x = 1$ and the Taylor polynomial of degree 6, centered at $x = 0$, for $y = \sin^2 x$ (see Exercise 26b) in order to get a polynomial representation of degree 6 for $y = \cos^2 x$.

WRITING & THINKING

28. Look up Brook Taylor and write a one-page report on his major accomplishments.

29. Determine the Taylor polynomial of degree 4, centered at $x = 0$, for the function $f(x) = 3 - 2x + 4x^2 - 2x^3$. Use your solution as a basis for a conjecture about the Taylor polynomial of a given polynomial function.

30. Differentiate the Taylor polynomial of degree 7 for $y = \sin t$ in order to get a polynomial approximation for $y = \cos t$ near $t = 0$. Compare your result with your answer to Exercise 3. Make a conjecture about derivatives of Taylor polynomial approximations to given functions.