Oresme used sums of areas of rectangles to represent the series in two different ways, as shown in Figures 2 and 3.

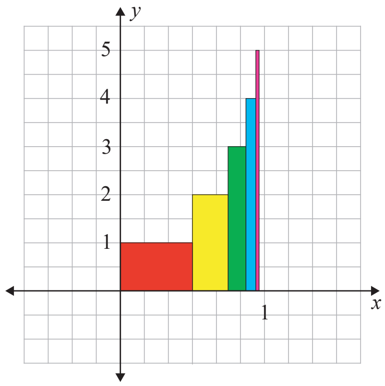

FIGURE 2

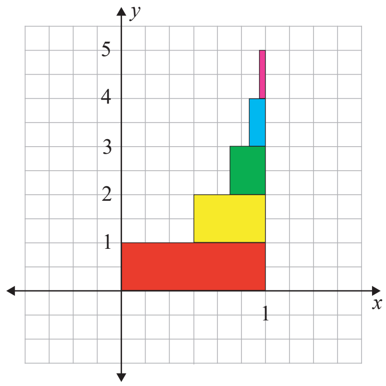

FIGURE 3

In Figure 2, the bases of the rectangles are the following decreasing powers of $2: 2^{-1}, 2^{-2}, 2^{-3}, \ldots$. The heights of the rectangles are the integers 1, 2, 3, …. Thus, the total area of all the rectangles is the sum $1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{8} + \ldots$. Notice that the area of each rectangle corresponds to a term in the original series whose sum we need to find.

In Figure 3, *the same total area* is decomposed into rectangles of height 1 with bases that are the following decreasing powers of 2: $2^0, 2^{-1}, 2^{-2}, 2^{-3}, \ldots$. Thus, the total area of all these rectangles is the sum $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$. We recognize this sum as a geometric series with $a = 1$ and $r = \frac{1}{2}$. Therefore, the total area is $\frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2.$

10.1 EXERCISES

PRACTICE

In Exercises 1–6, write the first five terms of the sequence.

1. $a_n = \frac{(-1)^n}{n}$

2. $a_n = \frac{3}{e^n}$

3. $a_n = \sqrt{2n+1}$

4. $a_n = \frac{\ln n}{\ln(n+1)}$

5. $a_n = \frac{n^2 - 1}{n+1}$

6. $a_n = \frac{n^3 + 1}{n+1}$

In Exercises 7–12, find a formula for the n^{th} term of the sequence, if one exists, assuming a domain starting with $n = 1$.

7. $\{3, -5, 7, -9, \ldots\}$

8. $\{11, 14, 17, 20, \ldots\}$

9. $\{5, 10, 15, 20, 25, \ldots\}$

10. $\{3, -2, 1, 0, 1, -2, \ldots\}$

11. $\{2, 5, 10, 17, 26, 37, 50, \ldots\}$

12. $\{3, 8, 15, 24, 35, 48, \ldots\}$

In Exercises 13–16, determine the limit $\lim_{n\to\infty} f(n)$.

13. $f(n) = 1 + \frac{3}{n}$

14. $f(n) = \frac{(-1)^n}{2}$

15. $f(n) = \frac{17n}{18n+1}$

16. $f(n) = \sum_{i=0}^{n} \frac{1}{2^i}$

In Exercises 17–22, compute the sums.

17. $\sum_{n=1}^{5} (2n-1)^2$

18. $\sum_{n=1}^{5} n(-1)^n$

19. $\sum_{n=4}^{8} (n-1)$

20. $\sum_{i=1}^{4} (3^i - 2^i)$

21. $\sum_{i=1}^{3} \frac{i^3 - 1}{i^2 + i + 1}$

22. $\sum_{i=1}^{5} \frac{i+1}{i^2 + i}$

In Exercises 23–30, find the sum if the series converges.

23. $\sum_{j=0}^{\infty} \left(\frac{\sqrt{2}}{2}\right)^j$

24. $\sum_{n=0}^{\infty} \frac{2n}{n+2}$

25. $\sum_{n=0}^{\infty} (-1)^{n+1} \left(\frac{1}{2}\right)^n$

26. $\sum_{n=0}^{\infty} \left(\frac{e}{\pi}\right)^n$

27. $\sum_{j=0}^{\infty} \frac{2j+1}{10j+1}$

28. $1 + e^{-1} + e^{-2} + e^{-3} + \cdots$

29. $4 - 0.4 + 0.04 - 0.004 + \cdots$

30. $1 - 2 + 3 - 4 + 5 - 6 + \cdots$

APPLICATIONS

31. A tennis ball drops from a height of one foot above the ground. It falls and bounces back to $\frac{1}{4}$ of its previous position. It continues to fall and bounce back in the same proportion until it comes to rest. (Take downward as a negative direction.)

a. What is the net distance traveled by the ball?
b. What is the total (absolute value) of the distance traveled by the ball?

32. The XYZ Corporation, a small business, generates \$300,000 in annual operating cash, which it spends on equipment, supplies, and services. Assume half of this amount is then spent by the recipients of XYZ's business in the general economy on similar items with similar results. Assuming this pattern of economic growth continues, how much total revenue is spent in the economy from the original \$300,000?

33. A patient takes a 10-milligram tablet of a blood pressure drug every day, and 30% of each day's dose is still in the patient's system after 24 hours. After a few weeks, how much of the medicine is in the patient's body at any one time (to the nearest milligram)?

34. A patient in a VA hospital takes a 5-milligram tablet of an experimental antidepressant drug daily. After 24 hours, a certain percentage of the previous day's dose is retained in the patient's system. After several weeks of taking the new medicine, the patient has 7.5 milligrams of the antidepressant in his body. What percentage of a daily dose is retained for the next day (to the nearest percent)?

35. Suppose 50 million taxpayers received a $1000 one-time tax cut, of which each person spent 90% and saved 10%. The proportion that each person spent is called the marginal propensity to consume and the proportion saved is the marginal propensity to save. Assume the money spent was then spent by others in the same proportion, and so on. How much total spending was generated by the tax cut?

WRITING & THINKING

36. **a.** Write out the first 10 partial sums to evaluate the expression $\sum_{n=1}^{10}(2n-1)$.

b. Use the results of part a. to determine a shortcut to evaluate $\sum_{n=1}^{100}(2n-1)$.

37. Determine a rational number equal to the number (repeating decimal) given.

a. $\frac{4}{10}+\frac{3}{100}+\frac{4}{1000}+\frac{3}{10{,}000}+\cdots=0.4343...$

b. $\sum_{j=0}^{\infty}\frac{5}{10^{2j}}$

38. Define a function $f(x)=1+x+x^2+x^3+\cdots=\sum_{j=0}^{\infty}x^j$. For each x in the interval $(-1, 1)$, the function is defined since the series converges for each x.

a. Determine a series for $f'(x)$. **Hint:** Differentiate term by term.

b. Define $G(x)$ by $G(x)=\frac{1}{1-x}$. Determine $G'(x)$.

c. Determine $f'(a)$ and $G'(a)$ for $a = 0.25, -0.25, -0.5$, and 0.5. Discuss or explain your answers in a short essay or paragraph.

39. Use long division of polynomials to confirm that

$$\frac{1}{1-r}=1+r+r^2+r^3+\cdots+r^n+\cdots.$$

40. Multiply $(1-r)(1+r+r^2+r^3+\cdots+r^n+\cdots)$ to confirm the result of Exercise 39.