Example 9: Using Special Products

Factor each of the following polynomials by using the list of special products.

a. $4x^2 - 25$ **b.** $x^2 + 12x + 36$ **c.** $x^3 - 125$

Solution

a. $4x^2 - 25$ is the difference of two squares: $4x^2 - 25 = (2x + 5)(2x - 5)$.

b. $x^2 + 12x + 36$ is a perfect square trinomial with $A = \frac{1}{2}(12) = 6$ and $A^2 = 6^2 = 36$:

$$x^2 + 12x + 36 = (x + 6)^2.$$

c. $x^3 - 125$ is the difference of two cubes with $A^3 = 5^3$:

$$x^3 - 125 = (x - 5)(x^2 + 5x + 25).$$

0.4 EXERCISES

PRACTICE

In Exercises 1–14, perform the indicated operation and simplify the expressions.

1. $\left(x+3x^2\right)+\left(5-x^2\right)$ **2.** $\left(x^2+2x-4\right)+\left(x^2-4\right)$

3. $\left(8a^2+5a+2\right)+\left(-3a^2+9a-4\right)$ **4.** $\left(3x^2+5x-4\right)+\left(2x^2-2x+4\right)$

5. $\left(2x^2+3x+8\right)-\left(x^2-5x+6\right)$ **6.** $\left(4x^3-7x^2+3x\right)-\left(-2x^3+5x-1\right)$

7. $\left(8x^2+9\right)-\left(4x^2-3x-2\right)$ **8.** $\left(y^3+4y^2-7\right)-\left(3y^3+y^2+2y+1\right)$

9. $\left(a^2-3ab+b^2\right)+\left(2a^2-5ab-b^2\right)$

10. $\left(7x^2-2xy+3y^2\right)+\left(-3x^2-2xy+5y^2\right)$

11. $\left(-3x^2-2xy+5y^2\right)-\left(4x^2+3xy\right)$

12. $\left(5x^2-3xy+7y^2\right)-\left(6x^2-9xy+8y^2\right)$

13. $2x^2\left(3x^2+5x-1\right)$ **14.** $-4y^2\left(2y^2+5y-4\right)$

Fill in the missing expressions in Exercises 15–17.

15. $(2x+3y)^2 = 4x^2 + \_\_\_\_ + 9y^2$

16. $(9x-5y)^3 = 729x^3 - 3(\_\_\_\_)x^2y + 3(\_\_\_\_)xy^2 - 125y^3$

17. $\left(3x^2+8y\right)^2 = 9(\_\_\_\_) + 48(\_\_\_\_) + 64(\_\_\_\_)$

In Exercises 18–33, find the products.

18. $(3x-8)(x-5)$ **19.** $(7x+6)(2x-3)$

20. $(5x+11)(3x-4)$

21. $(3x-4)(4x-3)$

22. $(3x+1)^2$

23. $(4x-3)^2$

24. $(7x-4y)^2$

25. $(3x+2y)^2$

26. $(4x-5)(4x+5)$

27. $(6x+y)(6x-y)$

28. $3x^2(1+3x)$

29. $2x(x^2+3x-4)$

30. $(x+2)(x^2-2x+4)$

31. $(x+3)(x^2-3x+9)$

32. $(y-5)(y^2+5y+25)$

33. $(x+2y)(x^2-2xy+4y^2)$

In Exercises 34–39, simplify each expression.

34. $5a+2(a-3)-(3a+7)$

35. $11+[3x-2(1+5x)]$

36. $3y-[5-7(y+2)-6y]$

37. $10t-[8-5(3-2t)-7t]$

38. $x(x-5)+[6x-x(4-x)]$

39. $x(2x+1)-[5x-x(2x+3)]$

Fill in the missing expressions in Exercises 40 and 41.

40. $11x^2-99y^2=11(x-\_\_\_\_)(x+\_\_\_\_)$

41. $16x^3+54y^3=2(2x+\_\_\_\_)(\_\_\_\_x^2-\_\_\_\_xy+9y^2)$

In Exercises 42–60, factor each expression completely. (Each factor should have integer coefficients.)

42. $x^2+6x-27$

43. $s^2-5s-14$

44. $x^2+27x+50$

45. $x^2+11x-26$

46. $2x^2-98$

47. $4b^3-64b$

48. $9y^3-16y$

49. $27a^2-12$

50. $x^2+6xy+9y^2$

51. x^6-1

52. x^5-x^3

53. $25x^8-16$

54. $125y^6-27z^3$

55. $2t^3+16y^3$

56. $1600x^2+880xy+121y^2$

57. s^4-1

58. $3a^2+12ab+12b^2$

59. $100xy^2+200xy+100x$

60. $x^{21}-x^{19}$

WRITING & THINKING

61. If you were teaching Algebra I to ninth grade students, how would you explain the difference between a variable and a constant in an algebraic expression?