Properties of Square Roots

In the following properties, a and b are positive real numbers.

1. $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$
2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

The squares of the positive integers are called **perfect square numbers**. Thus, the perfect squares are

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, and so on.

Expressions involving square roots are considered simplified if the radical contains no square factors. Thus $\sqrt{24}$ is not simplified because 24 has the square factor 4. We can simplify $\sqrt{24}$ as

$$\sqrt{24} = \sqrt{4 \cdot 6} = \sqrt{4} \cdot \sqrt{6} = 2\sqrt{6}.$$

CAUTION

Note the following common errors. These equations are *almost always false*.

Incorrect Equations

a. $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$

Let $a = 64$ and $b = 36$.

Then $\left.\begin{aligned}\sqrt{a+b} &= \sqrt{64+36} = \sqrt{100} = 10\\ \sqrt{a}+\sqrt{b} &= \sqrt{64}+\sqrt{36} = 8+6 = 14\end{aligned}\right\}$ are not equal.

b. $\sqrt{a^2+b^2} = a+b$

Let $a = 8$ and $b = 6$.

Then $\left.\begin{aligned}\sqrt{a^2+b^2} &= \sqrt{64+36} = \sqrt{100} = 10\\ a+b &= 8+6 = 14\end{aligned}\right\}$ are not equal.

c. $(a+b)^2 = a^2 + b^2$

Let $a = 6$ and $b = 8$.

Then $\left.\begin{aligned}(a+b)^2 &= (6+8)^2 = 14^2 = 196\\ a^2+b^2 &= 36+64 = 100\end{aligned}\right\}$ are not equal.

0.3 EXERCISES

PRACTICE

In Exercises 1–32, simplify each expression. Assume that all variables represent positive real numbers.

1. $64^{\frac{1}{3}}$
2. $144^{\frac{1}{2}}$
3. $16^{\frac{3}{4}}$
4. $(-8)^{\frac{2}{3}}$
5. $25^{-\frac{3}{2}}$
6. $4^{-\frac{5}{2}}$
7. $-81^{\frac{1}{4}}$
8. $-49^{\frac{3}{2}}$

9. $32^{\frac{2}{5}}$ **10.** $64^{-\frac{2}{3}}$ **11.** $x^2 \cdot x^{\frac{1}{2}}$ **12.** $x^3 \cdot x^{\frac{2}{3}}$

13. $x^{-\frac{1}{2}} \cdot x^{\frac{1}{3}}$ **14.** $x^{\frac{3}{4}} \cdot x^{\frac{1}{2}}$ **15.** $a^2 \cdot a^{-\frac{2}{5}}$ **16.** $x^{-3} \cdot x^{\frac{4}{3}}$

17. $\dfrac{x}{x^{\frac{2}{3}}}$ **18.** $\dfrac{t^2}{t^{\frac{1}{2}}}$ **19.** $\dfrac{x^{\frac{1}{4}}}{x^2}$ **20.** $\dfrac{y^{\frac{3}{4}}}{y^4}$

21. $\dfrac{b^{\frac{4}{5}}}{b^{\frac{1}{2}}}$ **22.** $\dfrac{x^{\frac{3}{2}}}{x^{-\frac{1}{3}}}$ **23.** $\dfrac{s^{\frac{3}{4}}}{s^{-\frac{1}{8}}}$ **24.** $\dfrac{x^{\frac{1}{2}}}{x^{\frac{4}{3}}}$

25. $\dfrac{x^{\frac{1}{2}} \cdot x^{-\frac{3}{4}}}{x^{-\frac{1}{2}}}$ **26.** $\dfrac{y^{\frac{2}{3}} \cdot y^{\frac{4}{3}}}{y^{-2}}$ **27.** $\dfrac{x^{\frac{3}{2}} \cdot x^{\frac{4}{5}}}{x^{-\frac{1}{2}} \cdot x^2}$ **28.** $\dfrac{x^{-\frac{2}{3}} \cdot x^{\frac{1}{2}}}{x \cdot x^{-\frac{3}{2}}}$

29. $\left(\dfrac{27x^3y^6}{x^{-3}}\right)^{\frac{1}{3}}$ **30.** $\left(\dfrac{16a^{-4}b^3}{ab^{-1}}\right)^{\frac{1}{4}}$ **31.** $\dfrac{(16s^2t)^{\frac{1}{4}}}{\left(5s^{\frac{1}{3}}t^{-\frac{1}{2}}\right)^2}$ **32.** $\dfrac{\left(25x^2y^{-1}\right)^{\frac{1}{2}}}{\left(5x^{\frac{1}{5}}y^{\frac{3}{5}}\right)^3}$

In Exercises 33–42, change each expression to an equivalent expression in radical notation. Assume that each variable expression represents a positive real number.

33. $y^{\frac{1}{2}}$ **34.** $x^{\frac{2}{5}}$ **35.** $x^{-\frac{1}{4}}$ **36.** $x^{-\frac{1}{2}}$

37. $(x-2)^{\frac{4}{5}}$ **38.** $(5a+3)^{\frac{2}{3}}$ **39.** $2\left(y^2+4\right)^{-\frac{1}{2}}$ **40.** $7\left(x^2+8\right)^{-\frac{1}{3}}$

41. $\dfrac{-2}{7}\left(t^2-4\right)^{-\frac{1}{4}}$ **42.** $\dfrac{5}{8}\left(x^2+6\right)^{-\frac{1}{2}}$

In Exercises 43–52, change each expression to an equivalent expression in exponential notation. Write each answer with no variables in a denominator. Assume that each variable expression represents a positive real number.

43. $\sqrt[4]{x}$ **44.** $\sqrt[3]{b}$ **45.** $\dfrac{1}{\sqrt[4]{s}}$ **46.** $\dfrac{1}{\sqrt[5]{x^2}}$

47. $\sqrt[3]{(y+10)^2}$ **48.** $\sqrt[4]{(x+1)^3}$ **49.** $\dfrac{1}{\sqrt[3]{a^2+2}}$ **50.** $\dfrac{2}{5\sqrt[3]{x^2-6}}$

51. $\dfrac{4x}{9\sqrt[3]{(x+5)^2}}$ **52.** $\dfrac{3y}{5\sqrt[3]{y^3+8}}$

In Exercises 53–60, simplify each radical expression. Assume that each variable represents a positive real number.

53. $\sqrt{48}$ **54.** $\sqrt{175}$ **55.** $\sqrt{\dfrac{9}{16}}$ **56.** $\sqrt{\dfrac{49}{225}}$

57. $\sqrt{150x}$ **58.** $\sqrt{112a}$ **59.** $\sqrt{108t^3}$ **60.** $\sqrt{216x^3}$