CHAPTER 5 PROJECT

Transit Planning and Revenue Analysis

You are an analyst intern at your city's transit authority. The authority uses mathematical models to forecast ridership growth, analyze fare elasticity, estimate operating costs, and plan long-term infrastructure investments. In this project, you will apply derivatives of logarithmic and exponential functions, growth and decay models, and elasticity of demand to support data-driven planning decisions.

1. Monthly ridership (in thousands of riders) is modeled by $R(t) = 250\ln(t+1) + 800$, where t is the number of months since a new route was launched. Find $R'(t)$ which represents the rate at which ridership is changing per month. Evaluate $R'(0)$ and $R'(12)$. Is ridership growing faster at launch or after one year? Explain.

2. A bus fleet depreciates in value according to $V(t) = 1,200,000e^{-0.12t}$, where t is measured in years and $V(t)$ is in dollars. Find $V'(t)$. What is the rate of depreciation at $t = 5$? After how many years will the fleet be worth less than \$400,000?

3. The transit authority models daily ridership as a function of fare price: $F(p) = 50,000e^{-0.04p}$, where p is the fare in dollars. Find $F'(p)$. Then calculate the elasticity of demand

$$E(p) = -\frac{pF'(p)}{F(p)}$$

and interpret your result.

4. Using the elasticity function from Question 3, evaluate $E(2.50)$ and $E(5.00)$. At each fare, classify demand as elastic, inelastic, or unit elastic. If the authority raises fares from \$2.50, will revenue increase or decrease? Explain using elasticity.

5. Daily revenue is given by $R(p) = p \cdot F(p) = 50{,}000\,pe^{-0.04p}$. Find $R'(p)$. Determine the fare that maximizes revenue. Verify that this fare corresponds to unit elasticity $(|E| = 1)$.

6. The operating cost function is $C(x) = 800{,}000 + 5x + 0.0001x^2$, where x is the number of daily riders. The average cost per rider is $A(x) = \dfrac{C(x)}{x}$. Evaluate the limits $\lim\limits_{x \to \infty} A(x)$ and $\lim\limits_{x \to 0^+} A(x)$, and interpret your results.