CHAPTER 4 PROJECT

Modeling Turtle Population Dynamics in a Conservation Study

You are a biology research assistant studying a population of native turtles in your university's nature preserve. The conservation team monitors population size, growth rates, and environmental conditions over time. You will use concavity and inflection points, the Second Derivative Test, curve sketching, absolute extrema, and optimization to analyze population trends and support conservation decisions.

1. The turtle population over a 12-week study is modeled by $P(t) = -0.5t^2 + 10t + 80,$ where t is time in weeks and $P(t)$ is the number of turtles. Find $P'(t)$ and $P''(t)$. Find the critical value, and use the Second Derivative Test to verify that it represents a maximum. Interpret the result: What is the peak population and when does it occur?

2. Algae growth in the preserve's pond (in g/m² per week) is modeled by $G(t) = -t^3 + 12t^2 - 36t + 100$ on $[0,10],$ where t is time in weeks. Find $G'(t)$ and $G''(t)$. Identify all critical values and the inflection point. Determine the intervals where the graph of $G(t)$ is concave up and concave down.

3. Sketch the graph of $G(t)$ on $[0,10]$. Label all critical values, the inflection point, and the endpoints. During which weeks is algae growth most concerning for turtle habitat quality?

4. Find the absolute maximum and absolute minimum values of $G(t)$ on [0, 10]. Based on your results, when should the conservation team treat the pond to control algae growth?

5. After a nutrient runoff event, the dissolved oxygen level in the pond (mg/L) is modeled by $D(t)=\frac{40t}{t^2+16}$, where t is time in weeks. Find $D'(t)$. Determine the maximum dissolved oxygen level. Sketch the graph of $D(t)$, and identify the horizontal asymptote. Interpret what the horizontal asymptote means in this context.

6. The conservation team is building a rectangular nesting enclosure along a riverbank, with the side next to the river left open. A total of 200 feet of fencing is available. Find the dimensions that maximize the enclosed area. If each turtle nesting site requires 25 square feet, how many nesting sites can be supported?

7. The team designs a cylindrical turtle observation shelter enclosing 200 cubic feet. Flooring costs \$6/ft², wall material costs \$4/ft², and the open-mesh roof costs \$2/ft². Find the dimensions that will minimize total construction cost and the minimum cost of the shelter.

8. To model the long-term population behavior beyond the study, an ecology professor proposes the following rational model:

$$P(t) = \frac{500t}{t+5},$$

where $t \geq 0$ is measured in months and $P(t)$ is the turtle population. Find $P'(t)$ and $P''(t)$. Determine where the graph of $P(t)$ is increasing, decreasing, concave up, and concave down. Interpret your results in the context of population growth. Explain what population level the model approaches as t becomes large, and what this implies for long-term conservation.

9. At the end of the study, a colleague states that the turtle population "has peaked and is now declining," based on the quadratic model from Question 1. Using the rational model from Question 8, explain why a deceleration in growth rate does not mean a population is declining. Which population model should the conservation team use for long-term planning—the quadratic or the rational model? Justify your answer using mathematical and biological reasoning.