CHAPTER 4 PROJECT

Optimizing Stadium Operations

You are a data analytics intern for your university's athletics department. With football season approaching, the department needs quantitative guidance on ticket pricing, concessions staffing, merchandise layout, and parking logistics. You will use concavity and inflection points, the Second Derivative Test, curve sketching, and optimization to guide business decisions.

1. Game-day ticket revenue (in thousands of dollars) is modeled by

$$R(p) = -0.8p^2 + 64p - 400,$$

where p is the ticket price (in dollars). Find $R'(p)$ and $R''(p)$. Find the critical value and use the Second Derivative Test to verify that it corresponds to a maximum. Determine the ticket price that maximizes revenue and the resulting maximum revenue.

2. Concession profit during a football game (in thousands of dollars) is modeled by

$$P(t) = -t^3 + 9t^2 - 15t + 50,$$

where t is the number of hours after the stadium gates open ($0 \le t \le 6$). Find $P'(t)$ and $P''(t)$. Locate the inflection point. What is happening to concession profit at the inflection point?

3. Using the function $P(t)$ from Question 2, find and classify each critical value using the First Derivative Test. Determine the intervals where the graph of $P(t)$ is increasing, decreasing, concave up, and concave down. Sketch the graph of $P(t)$.

4. Find the absolute maximum and absolute minimum values of $P(t)$ on $[0,6]$. At what time should the concessions manager schedule the greatest number of workers? When is concession profit at its lowest, and what is the minimum profit amount?

5. The rate at which vehicles enter the football stadium parking lot (in vehicles per minute) is given by

$$F(x) = \frac{800x}{x^2 + 400}$$

where x is the percentage of parking lot capacity filled, with $0 \le x \le 100$. Find the critical value of $F(x)$ on its domain and determine whether it represents a maximum or minimum. What is the maximum traffic flow rate and at what capacity level does it occur? Explain, in context, why traffic flow begins to decline as the parking lot becomes more full.

6. The athletics department has 120 feet of portable fencing to create a rectangular merchandise display against a stadium wall. Only three sides require fencing. Determine the dimensions that maximize the display area. Use the Second Derivative Test to verify that your solution yields a maximum. Find the maximum possible display area.

7. Ticket profit per game is modeled by

$$P(p) = (2000 - 25p)(p - 15) - 12{,}000,$$

where p is the ticket price and $2000 - 25p$ represents the number of tickets sold. Expand the profit function. Find the ticket price that maximizes profit. Determine the maximum profit and the number of tickets sold at that price.

8. To improve public relations for the football program, a newly appointed athletics department director mandates that at least 1200 tickets must be sold per game. Does the profit-maximizing ticket price from Question 7 satisfy this sales requirement? If not, determine the ticket price that maximizes profit subject to the minimum-sales constraint. What is the resulting profit under this constraint? How much profit does the ticket-sales mandate cost the department per game?

9. The department must design a cylindrical tank that holds 500 gallons (approximately 66.84 cubic feet) of drinking water for tailgating. The material for the top and bottom of the tank costs $8 per square foot, and the side material costs $5 per square foot. Find the dimensions that minimize the total cost of the water tank. Verify your result using the Second Derivative Test. Determine the minimum possible cost.

10. The sales manager states: "We should set ticket prices to maximize revenue, not profit." Using your results from Questions 1 and 7, explain why the revenue-maximizing and profit-maximizing prices differ. Calculate the profit when tickets are priced to maximize revenue and compare it to the maximum profit. What are the consequences of following the manager's advice? Under what circumstances, if any, would maximizing revenue be the right goal?