CHAPTER 3 PROJECT

Modeling Fox Population Dynamics

You are a field research assistant for an ecology professor studying a population of foxes in a state nature reserve. The team tracks population size, birth and death rates, and the effects of environmental carrying capacity. You will use the Product and Quotient Rules, the General Power Rule, implicit differentiation, analysis of increasing and decreasing intervals of a function, the First Derivative Test, and absolute extrema to model and predict population behavior.

1. The fox population (in tens) at time t (years) is modeled by $N(t) = (t^2 + 4)(20 - t)$ for $0 \le t \le 20$. Find $N'(t)$. Simplify your result completely by expanding and combining like terms.

2. The average food supply per fox is modeled by

$$F(N) = \frac{500 - 2N}{N + 10},$$

where N is the fox population (in tens). Find $F'(N)$. Is F increasing or decreasing as the population grows? What does this indicate ecologically?

3. The carrying capacity is the maximum population that can be supported in a given area. For the foxes in the nature reserve, the proportion of the carrying capacity that is filled at time t (years) is modeled by $P(t) = \sqrt{t^2 + 9} - 3$, for $t \ge 0$. Find $P'(t)$. Evaluate $P'(0)$ and $P'(4)$. Based on these values, determine whether the rate at which the capacity is being filled is speeding up or slowing down.

4. Using $N(t)$ from Question 1, determine the intervals on $[0,20]$ where the fox population is increasing and decreasing. Identify all critical value(s) and classify them using the First Derivative Test. In what year does the fox population reach its maximum?

5. The birth rate per fox is given by $b(N) = 3.0 - 0.005N$, and the death rate per fox is modeled by $d(N) = 0.5 + 0.003N$, both measured per year, where N is the population (in tens). The net per capita growth rate is defined by $r(N) = b(N) - d(N)$. Find $r(N)$, and then compute $r'(N)$. At what population level does net growth equal zero (the equilibrium)? Is this equilibrium stable? Explain.

6. Total population growth is given by $G(N) = N \cdot r(N)$, where N and $r(N)$ are as defined in Question 5. Find $G'(N)$ and determine the population size that maximizes total growth. Evaluate G at that population level. Explain why the population that maximizes growth is not the same as the equilibrium population.

7. A prey species has population (in tens) modeled by

$$Q(t) = \frac{200t}{t^2 + 25},$$

Find $Q'(t)$ and determine the time at which the prey population peaks. What is the maximum prey population? Discuss how this peak relates to the fox population model.

8. The fox and prey populations (both measured in tens) satisfy the implicit relationship $F^2 + 2FQ - Q^2 = 800$. Use implicit differentiation to find $\frac{dF}{dQ}$. Verify that $F = 20$ and $Q = 20$ satisfy the equation, and then evaluate $\frac{dF}{dQ}$ at that point. Interpret the meaning of this rate of change.

9. The reserve manager wants to keep the fox population between 20 and 50 (tens) to maintain ecological balance between the fox and prey populations. Using $N(t) = (t^2 + 4)(20 - t)$, where N is the fox population (in tens) after t years, evaluate $N(0)$, $N(5)$, $N(10)$, $N(15)$, and $N(20)$. Find the absolute maximum of N on $[0, 20]$. During what time interval is the population within the desired range of 20 to 50 (tens)?

10. The ecology professor says, "We should manage the fox population to maximize its growth rate." Using your analysis from Questions 5 and 6, explain the difference between maximizing the per capita growth rate $r(N)$ and maximizing the total population growth $G(N)$ which occurs at a smaller population size? Finally, discuss why a wildlife manager might prefer to maintain the population near the equilibrium level from Question 5 rather than at population that maximizes growth from Question 6. How do stability and long-term sustainability factor into this decision?