CHAPTER 2 PROJECT

Modeling Drug Absorption in a Pharmacology Study

You are a research assistant in your university's pharmacology lab. The lab studies how oral medications are absorbed into the bloodstream over time. In this project, you will use limits, continuity, average and instantaneous rates of change, the limit definition of the derivative, the Power Rule and other basic differentiation rules, and marginal analysis to model drug concentration, absorption rates, and dosing effectiveness.

1. A pain medication's plasma concentration (mg/L) is modeled by the piecewise function

$$C(t)=\begin{cases}5t & \text{if } 0\le t<2\\ 10 & \text{if } 2\le t<6\\ 10-1.5(t-6) & \text{if } 6\le t\le 12\end{cases}$$

where t is the time (in hours) since the drug was administered. Find the one-sided limits $\lim_{t\to 2^-} C(t)$ and $\lim_{t\to 2^+} C(t)$. Is C continuous at $t = 2$? Interpret the meaning of this result in terms of the drug's concentration in the bloodstream.

2. Evaluate $\lim_{t\to 6^-} C(t)$ and $\lim_{t\to 6^+} C(t)$ Is C continuous at $t = 6$? Sketch the graph of $C(t)$. Describe the three phases of the concentration profile using appropriate pharmacological terminology (absorption, plateau, and elimination).

3. The total amount of drug absorbed (in mg) by time t is modeled by $A(t)=8t^2-0.5t^3$, for $0 \le t \le 12$. Calculate the average rate of absorption between $t = 1$ to $t = 4$. Interpret the result in context.

4. Using the limit definition of the derivative, the instantaneous rate of absorption at $t = 3$ is given by

$$A'(3) = \lim_{h \to 0} \frac{A(3+h) - A(3)}{h},$$

where $A(t)$ is defined as in Question 3. Evaluate this limit step by step, showing all algebraic work.

5. Differentiate $A(t) = 8t^2 - 0.5t^3$ using the Power Rule and then evaluate the derivative at $t = 3$. Verify that your result for $A'(3)$ agrees with your answer from Question 4. Determine the time at which the absorption rate reaches zero. Interpret this result in the context of the drug absorption process.

6. The drug's effectiveness is modeled by $E(c) = 20c - c^2$, where c is the plasma concentration in mg/L and E is an effectiveness score. Find $E'(c)$ and then evaluate $E'(2)$ and $E'(10)$. Interpret these values in the context of increasing the drug concentration from $c = 2$ and $c = 10$.

7. The cost of producing x doses is $C(x) = 0.002x^2 + 3x + 500$. The hospital receives \$8 in revenue for each dose sold. Determine the profit function $P(x)$ and the marginal profit function $P'(x)$. Find the profit-maximizing production level by solving the equation $P'(x) = 0$. What is the marginal cost at that production level, and why is this result economically significant?

8. Two formulations of the same drug are being tested. The plasma concentration (mg/L) of each formulation is modeled by

$$C_A(t) = \frac{12t}{t+2} \quad \text{and} \quad C_B(t) = \frac{15t}{t+5},$$

where t is the time (in hours) since the drug was administered. Compute the limit of each function as $t \to \infty$. Which formulation approaches a higher long-run concentration? Evaluate each function at $t = 1$. Based on your calculations, which formulation has the higher concentration one hour after being administered?

9. A colleague claims, "Higher drug concentration always means better treatment." Using the effectiveness model $E(c)$ and its derivative from Question 6, explain why this claim is misleading. What happens to the marginal benefit of increasing the drug concentration as the plasma concentration c increases? How can a pharmacologist use the derivative $E'(c)$ to help identify an efficient dosing strategy?