CHAPTER 2 PROJECT

Dynamic Pricing at a Ride-Share Company

You are a pricing analytics intern at a ride-share company. The company uses mathematical models to set surge pricing, forecast demand, and analyze revenue. In this project, you will apply one-sided and two-sided limits, continuity, average and instantaneous rates of change, the limit definition of the derivative, the Power Rule and other basic differentiation rules, and marginal analysis to inform management decisions.

1. The surge multiplier during a concert is modeled by the piecewise function

$$S(t)=\begin{cases}1.0 & \text{if } 0\le t<2\\ 0.5t+0.5 & \text{if } 2\le t<4\\ 2.5 & \text{if } 4\le t\le 6\end{cases}$$

where t is the number of hours after the concert starts.

a. Find the one-sided limits $\lim\limits_{t\to 2^-} S(t)$ and $\lim\limits_{t\to 2^+} S(t)$.

b. Does $\lim\limits_{t\to 2} S(t)$ exist?

c. Is S continuous at $t = 2$? Justify your answer.

2. Evaluate $\lim\limits_{t\to 4^-} S(t)$ and $\lim\limits_{t\to 4^+} S(t)$. Is S continuous at $t = 4$? Sketch the graph of $S(t)$, clearly indicating any discontinuities. Interpret the practical meaning of each discontinuity for a rider opening the app at that exact time.

3. The total number of ride requests (in hundreds) during the first t hours is modeled by

$$N(t) = 3t^2 + 2t.$$

Calculate the average rate of change of $N(t)$ from $t = 1$ to $t = 3$. Interpret your result in the context of ride demand.

4. Using the limit definition of the derivative, the instantaneous rate of change of $N(t)$ at $t = 2$ is given by

$$N'(2) = \lim_{h \to 0} \frac{N(2+h) - N(2)}{h}.$$

Evaluate this limit step by step, showing all algebraic work. Compare the resulting value with the average rate of change found in Question 3.

5. Differentiate $N(t) = 3t^2 + 2t$ using the Power Rule and then evaluate the derivative at $t = 2$. Verify that your result for $N'(2)$ matches your answer from Question 4. Evaluate $N'(0)$ and $N'(5)$. Explain why the instantaneous rate of change increases over time based on the form of the derivative.

6. The daily revenue (in dollars) from a pricing zone is modeled by

$$V(p) = 500p - 20p^2,$$

where p is the price per mile. Find $V'(p)$. Find the price p that maximizes revenue by solving the equation $V'(p) = 0$. What is the maximum revenue?

7. The daily cost (in dollars) is given by

$$C(x) = 0.01x^2 + 2x + 800,$$

where x is the number of rides per day. Determine the marginal cost function $C'(x)$. Evaluate $C'(100)$ and $C'(300)$. Interpret these values.

8. If the daily revenue and cost (in dollars) are given by

$$R(x) = 15x - 0.005x^2 \quad \text{and} \quad C(x) = 0.01x^2 + 2x + 800,$$

determine the profit function $P(x)$ and the marginal profit function $P'(x)$. Find the number of daily rides x that maximizes profit by solving the equation $P'(x) = 0$. What is the marginal profit at this value of x, and why does this result make economic sense?

9. A company executive claims, "If marginal revenue is positive, we should always increase the number of rides." Using the model from Question 8, explain why this statement is incorrect. What is the correct criterion for deciding whether to increase the number of rides? At the profit-maximizing output, what relationship must hold between marginal revenue and marginal cost? Explain why marginal profit, not marginal revenue, is the appropriate quantity to monitor.