CHAPTER 1 PROJECT

Studying Water Quality in a Campus Watershed

You are a research assistant for an environmental science team monitoring the water quality of a stream that flows through your university's campus. The team tracks dissolved oxygen levels, pollutant concentrations, bacteria counts, and temperature across seasons. In this project, you will use polynomial and rational models to analyze the data and advise the university's sustainability office.

1. The dissolved oxygen level in the stream (mg/L) depends on water temperature (°C) and is modeled by $D(T) = -0.2T + 14$, valid for water temperatures between 5 °C and 30 °C. State the domain and range of D using interval notation. Evaluate $D(10)$ and $D(25)$. Which temperature produces a higher dissolved oxygen level? Why does this make biological sense?

2. The daily bacteria count (in thousands per mL) in the stream after a rainfall event is modeled by $B(t) = 3t^2 - 24t + 85$, where t is days since the rainfall. Evaluate $B(0), B(4)$, and $B(8)$. The graph of this quadratic function is a parabola; determine whether the parabola opens upward or downward, and interpret the shape of the graph in terms of the bacteria count over time.

3. Find the vertex (the lowest point on the graph) of $B(t) = 3t^2 - 24t + 85$ and interpret both coordinates.

4. The concentration of a chemical pollutant downstream from a campus parking lot is modeled by:

$$P(d) = \frac{120}{d+3}$$

where d is distance downstream in meters and P is concentration in mg/L. Evaluate $P(0), P(7), P(27)$, and $P(97)$. What happens to P as d gets very large? Identify the horizontal asymptote and explain what it means for the stream's ability to dilute pollutants over distance.

5. Seasonal nitrate levels (mg/L) are modeled by the following polynomial.

$$N(m) = 0.1m^3 - 2.1m^2 + 12m + 5, \text{ for } m = 1 \text{ through } m = 12$$

Evaluate $N(m)$ for $m = 1, 3, 5, 7, 9$, and 12. During which months are nitrate levels above 15 mg/L? Describe the seasonal pattern and suggest an environmental explanation.

6. The rate at which the stream naturally processes pollutants (mg removed per hour) is modeled by

$$R(c) = \frac{50c}{c+8}$$

where c is the current pollutant concentration in mg/L. Evaluate $R(2), R(8), R(40)$, and $R(200)$. Using a graphing calculator, graph the function R and determine the value that R approaches as c increases without bound. Explain what this biological ceiling means: Why can't the stream process pollutants faster than this limit, no matter how polluted the water becomes?

7. After a sewage leak, the team compares two models for E. coli counts (thousands per mL, t in hours):

Model A: $E_A(t) = -5t^2 + 40t + 100$

Model B: $E_B(t) = \dfrac{800}{t+2}$

Evaluate both at $t = 0, 4, 8$, and 16. Which model predicts that E. coli eventually reaches zero (or negative)? Which predicts a persistent low-level presence? Which behavior is more biologically realistic, and why?

8. The sustainability office asks for your overall assessment. You have used linear, quadratic, polynomial, and rational functions across your analyses. Which type of function was most useful for modeling short-term events (like a pollutant spike after rainfall)? Which was most useful for modeling long-term or distance-based processes (like dilution downstream)? Identify one place in your analysis where the model clearly breaks down. What does this teach you about the relationship between mathematical models and natural systems?