8.4 Exercises

Basic Concepts

1. Describe the connection between the z-transformation and the standard normal random variable.

Exercises

2. Determine the z-value given $\mu = 15$, $\sigma = 2$, and $x = 19$. Indicate where the z-value would be on the standard normal distribution.

3. Determine the z-value given $\mu = 0.023$, $\sigma = 0.001$, and $x = 0.020$. Indicate where the z-value would be on the standard normal distribution.

4. The lengths of full-term pregnancies have a normal distribution with a mean of 266 days and a standard deviation of 16 days.

- **a.** Find the probability that the length of a pregnancy is between 250 and 282 days.
- **b.** Find the probability that the length of a pregnancy is greater than 275 days.
- **c.** Find the probability that the length of a pregnancy is less than 250 days.

5. If too much glucose binds to hemoglobin in the blood, this is an indicator of diabetes. A test called A1c measures the amount of glucose that binds to hemoglobin. A1c level in a nondiabetic person has a normal distribution with a mean of 5% and a standard deviation of 0.5%.

- **a.** Find the probability that the A1c level is between 5.5% and 5.7%.
- **b.** Find the probability that the A1c level is greater than 5.7%.
- **c.** Find the probability that the A1c level is less than 5.2%.

6. According to the Bureau of Labor Statistics, the average data scientist salary is \$100,560.[3] If the standard deviation of data scientist incomes is \$25,000 and we assume that salaries are normally distributed, what percentage of data scientists earn more than \$150,000?

7. A certain component for the newly developed electronic diesel engine is considered to be defective if its diameter is less than 8.0 mm or greater than 10.5 mm. The distribution of the diameters of these parts is known to be normal with a mean of 9.0 mm and a standard deviation of 1.5 mm. If a component is randomly selected, what is the probability that it will be defective?

8. A television manufacturer is studying television remote control unit usage. One of the criteria they are measuring is the distance at which people attempt to activate the television set with the remote unit. They have discovered that activation distances are normally distributed with an average activation distance of six feet with a standard deviation of three feet. If a remote unit's maximum range is ten feet, what proportion of the time will users attempt to operate the remote outside of the operating limit?

9. According to the Bureau of Labor Statistics, the mean weekly earnings for people working in a sales related profession in 2022 was \$938.[4] Assume that the weekly earnings are approximately normally distributed with a standard deviation of \$100.

a. What are the mean weekly earnings for people working in a sales related profession in 2022?

b. If a salesperson was randomly selected, find the probability that their weekly earnings exceed \$1000.

c. If a salesperson was randomly selected, find the probability that their weekly earnings are at most \$800.

d. If a salesperson was randomly selected, find the probability that their weekly earnings are between \$800 and \$950.

e. Do you feel that it is reasonable to assume that the weekly earnings have a normal distribution? Why or why not?

10. The repair time for air conditioning units is believed to have a normal distribution with a mean of 38 minutes.

a. What is the standard deviation of repair time if 40% of the units are repaired between 33 and 43 minutes?

b. Using the value of the standard deviation that you computed in **a.**, what is the probability that a repair will be longer than an hour?

c. Using the value of the standard deviation that you computed in **a.**, what is the probability that the repair time for an air conditioning unit will be less than 25 minutes?

11. LED monitors manufactured by TSI Electronics have life spans which have a normal distribution with an average life span of 100,000 hours and a standard deviation of 15,000 hours. If an LED monitor is selected at random, find the following probabilities.

a. The probability that the life span of the monitor will be less than 90,000 hours.

b. The probability that the life span of the monitor will be more than 120,000 hours.

c. The probability that the life span of the monitor will be between 100,000 hours and 120,000 hours.

12. A beer distributor believes the amount of beer in a 12-ounce can of beer has a normal distribution with a mean of 12 ounces and a standard deviation of 1 ounce. If a 12-ounce beer can is randomly selected, find the following probabilities.

a. The probability that the 12-ounce can of beer will actually contain less than 11 ounces of beer.

b. The probability that the 12-ounce can of beer will actually contain more than 12.5 ounces of beer.

c. The probability that the 12-ounce can of beer will actually contain between 10.5 and 11.5 ounces of beer.

13. A statistics teacher believes that the final exam grades for her business statistics class have a normal distribution with a mean of 82 and a standard deviation of 8.

a. Find the score which separates the top 10% of the scores from the lowest 90% of the scores.

b. The teacher plans to give all students who score in the top 10% of scores an A. Will a student who scored a 90 on the exam receive an A? Explain.

c. Find the score which separates the lowest 20% of the scores from the highest 80% of the scores.

d. The teacher plans to give all students who score in the lowest 10% of scores an F. Will a student who scored a 65 on the exam receive an F? Explain.

14. In order for you to become a member of Mensa, a worldwide organization with approximately 145,000 members, your IQ score must be in the top 2%. The word *mensa* is Latin for "table," and was chosen to denote a group or round table of people with equal ability. In 1996, Mensa, which was founded by two British barristers, celebrated its 50^{th} birthday. American Mensa Ltd., which was founded in 1960 has more than 57,000 members. Assuming that IQ scores have an approximately normal distribution with a mean and standard deviation of 100 and 15, respectively, answer the following questions.

a. What IQ must one have in order to become a member of Mensa?

b. What percent of all Americans have an IQ of at least 145?

c. What percent of all members of Mensa have an IQ of at least 145?

d. If Mensa decided to become more exclusive, and accepted only the top 1% instead of the top 2% as members, what IQ would one need in order to become a member of Mensa?

15. A farmer believes that the yields of his tomato plants have a normal distribution with an average yield of 10 lb. and a standard deviation of 2 lb. The farmer would like to identify the plants which yield the highest 5% and save them for breeding purposes.

a. Compute the yield which separates the highest 5% of yields from the lowest 95% of yields.

b. If a tomato plant yielded 14 lb. would it be kept for breeding purposes? Explain.

c. If a tomato plant yielded 13 lb. would it be kept for breeding purposes? Explain.

8.5 Assessing Normality

Many of the statistical tests that are discussed in this book require that the data be a simple random sample from a population that has a *normal* distribution, or is at least approximately normal. If a histogram of the data is symmetric and bell-shaped, we can assume normality. However, the shape of a histogram can be hard to determine with a small sample of data. Therefore, we need additional ways to assess normality. One of these alternative methods is called a **normal probability plot** (or **normal quantile plot**).