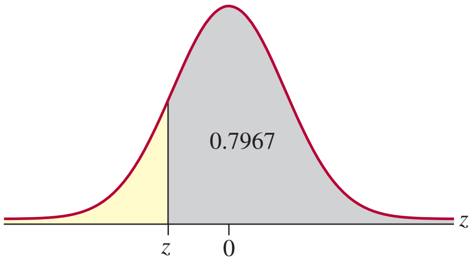
Please note that the value of z is to the left of 0. Thus, the value of z is going to be negative. Note that the area to the left of z represents the cumulative probability (in the above figure). So, to find the value of z, we only need to find 0.1469 in the body of Appendix A, Table A. The value of z with the area 0.1469 to the left of it is -1.05. That is, $P(z < -1.05) = 0.1469$.

d. Once again, a picture can be very helpful.



Note that from the picture, we have the area to the right of z. However, we know that the total area under the curve is 1. Thus, if the area to the right of z is 0.7967, then the area to the left of z is $1 - 0.7967 = 0.2033$. From the picture, it is clear that if we find 0.2033 in the body of Appendix A, Table A, the corresponding value of z is the value we are interested in. This value of z is -0.83. Therefore, the value of z with the area 0.7967 to the right is -0.83.

8.3 Exercises

Basic Concepts

1. When we say a random variable has a distribution, what does that mean?
2. Why is the standard normal distribution called the standard normal?
3. Would it be unusual for a standard normal distribution to have an observation greater than 6?
4. Would it be unusual for a standard normal distribution to have an observation less than -4?
5. What is the standard normal distribution? What are the parameters of the distribution?
6. Why is the standard normal distribution important?

Exercises

7. What proportion of the area under the standard normal curve falls between the following z-values?

   a. 0 and 0.67

   b. 0 and 1.645

   c. 0 and 1.96

   d. 0 and 2.575

8. What proportion of the area under the standard normal curve falls between the following z-values?

   a. -0.67 and 0   **c.** -1.96 and 0

   b. -1.645 and 0   **d.** -2.575 and 0

9. What proportion of the area under the standard normal curve falls between the following z-values?

   a. -0.85 and 0.85   **c.** -1.56 and 1.98

   b. -0.55 and 0.55   **d.** -2.23 and 2.96

10. What proportion of the area under the standard normal curve falls between the following z-values?

   a. -0.97 and 0.97   **c.** -1.95 and 2.28

   b. -0.54 and 1.82   **d.** -2.89 and 1.59

11. Using the standard normal tables in Appendix A, determine the following probabilities and draw the corresponding diagram..

   a. $z \le 0$ **d.** $z \le 1$

   b. $z \ge 0$ **e.** $z \ge -1$

   c. $z \le -1$ **f.** $z \ge 1$

12. Using the standard normal tables in Appendix A, determine the following probabilities and draw the corresponding diagram.

   a. $z \le -0.44$ **d.** $z \le -0.67$

   b. $z \ge 0.44$ **e.** $z \ge 0.67$

   c. $-0.44 \le z \le 0.44$ **f.** $-0.67 \le z \le 0.67$

13. Using the standard normal tables in Appendix A, determine the following probabilities and draw the corresponding diagram.

   a. $z \le -1.28$ **d.** $z \le -1.96$

   b. $z \ge 1.28$ **e.** $z \ge 1.96$

   c. $-1.28 \le z \le 1.28$ **f.** $-1.96 \le z \le 1.96$

14. Using the standard normal tables in Appendix A, determine the following probabilities and draw the corresponding diagram.

   a. $P(0 \le z \le 0.79)$ **c.** $P(z \ge 1.89)$

   b. $P(-1.57 \le z \le 2.33)$ **d.** $P(z \le -2.77)$

15. Using the standard normal tables in Appendix A, determine the following probabilities and draw the corresponding diagram.

   a. $P(0 \le z \le 1.24)$ **c.** $P(z \ge 3.22)$

   b. $P(-2.64 \le z \le 3.32)$ **d.** $P(z \le -3.39)$

Note

We recommend drawing the corresponding areas on a normal curve when working on these exercises.

16. Find the value of z such that 0.05 of the area under the curve lies to the right of z and draw the corresponding diagram.

17. Find the value of z such that 0.01 of the area under the curve lies to the right of z and draw the corresponding diagram.

18. Find the value of z such that 0.10 of the area under the curve lies to the right of z.

19. Find the value of z such that 0.05 of the area under the curve lies to the left of z and draw the corresponding diagram.

20. Find the value of z such that 0.01 of the area under the curve lies to the left of z.

21. Find the value of z such that 0.10 of the area under the curve lies to the left of z.

22. Find the value of z such that 0.7458 of the area under the curve lies between $-z$ and z and draw the corresponding diagram.

23. Find the value of z such that 0.9505 of the area under the curve lies between $-z$ and z.

24. Find the value of z such that 0.90 of the area under the curve lies between $-z$ and z.

Note

We recommend drawing the corresponding areas on a normal curve when working on these exercises.

8.4 Applications of the Normal Distribution

Most normal distributions of real data do not have a mean of zero and standard deviation of one. However, we can perform a transformation to standardize any normal random variable into a standard normal distribution.

Standardizing a Normal Random Variable

The following formula can transform any normal random variable into a standard normal random variable, z:

$$z = \frac{x - \mu}{\sigma}$$

where x is a normal random variable with mean μ and standard deviation σ.

FORMULA

If we look at the individual pieces, exactly how the transformation works is not very mysterious. First, the numerator $x - \mu$ centers the z-distribution around zero. By subtracting the mean of the random variable from each data value, the mean of the resulting random variable will be zero. A short example illustrates the point.