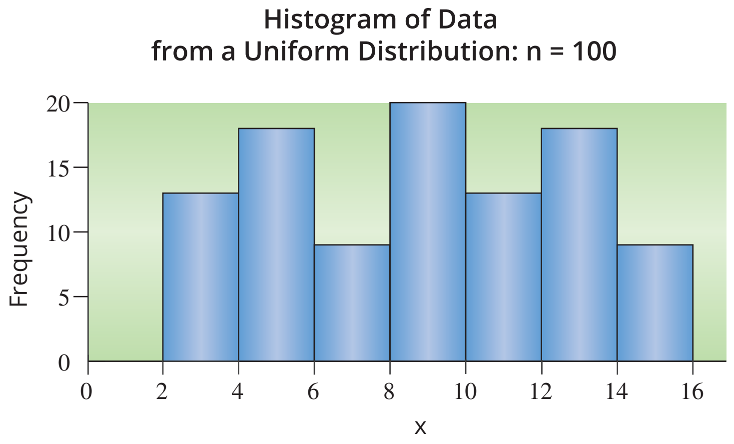

Figure 8.1.2

However, if we were to generate 1000 observations, the distribution would likely begin to level (see Figure 8.1.3).

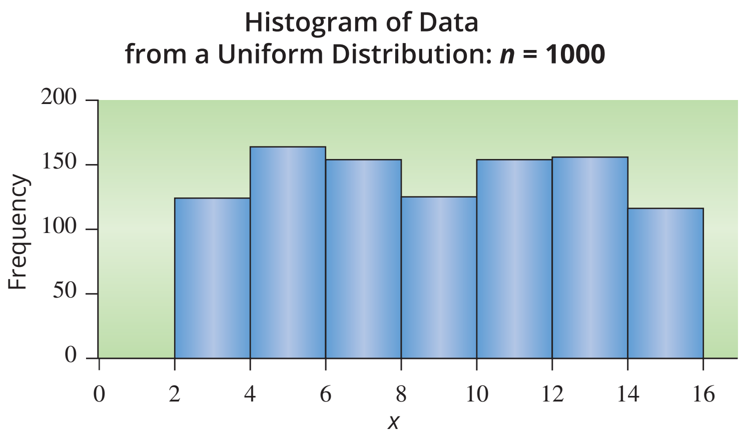

Figure 8.1.3

This idea is similar to the law of large numbers we discussed in Chapter 6 which states that a relative probability approaches the classical probability when enough trials are done. In the case of a random variable's probability distribution, a larger sample size should produce a distributional shape closer to the expected shape.

8.1 Exercises

Basic Concepts

1. Probability is defined differently for discrete and continuous random variables. Describe this difference.
2. How is the continuous uniform distribution different from the discrete uniform distribution?
3. What is the uniform probability density function?
4. Describe the shape of the density function for a uniform distribution.

Exercises

5. Suppose a continuous random variable is uniformly distributed between 10 and 70.

a. What is the mean of the distribution?

b. What is the standard deviation of the distribution?

c. What is the probability that a randomly selected value will be above 45?

d. What is the probability that a randomly selected value will be less than 30?

e. What is the probability that a randomly selected value will be between 25 and 50?

f. Find the probability that a randomly selected value will exactly equal 35.

6. A 14-volt lawn mower battery actually has a voltage that is uniformly distributed between 13.3 and 14.5 volts.

a. What is the mean of the distribution?

b. What is the standard deviation of the distribution?

c. What is the probability that a particular battery has a voltage above 14.0 volts?

d. What is the probability that a particular battery has a voltage less than 13.6 volts?

e. What is the probability that a particular battery has a voltage between 14.0 and 14.3 volts?

f. Find the probability that a particular battery has a voltage that is exactly 14.2 volts.

7. Polar Bear Frozen Foods manufactures frozen French fries for sale to grocery store chains. The final package weight is thought to be a uniformly distributed random variable. Assume X, the weight of French fries, has a uniform distribution between 57 ounces and 63 ounces.

a. What is the mean weight for a package?

b. What is the standard deviation for the weight of a package?

c. What is the probability that a store will receive a package weighing less than 59 ounces?

d. What is the probability that a package will contain between 60 and 63 ounces?

e. What is the probability that a package will contain more than 62 ounces?

f. Find the probability that a package will contain exactly 60 ounces.

8. The annual growth in height of cedar trees is believed to be distributed uniformly between 6 and 11 inches.

a. Draw a picture of the distribution of growth in height of cedar trees.

b. What is the mean growth per year?

c. What is the standard deviation of the growth per year?

d. What is the probability that a randomly selected cedar tree will grow between 9 and 10 inches in a given year?

e. Find the probability that a randomly selected cedar tree will grow less than 8 inches in a given year.

f. Find the probability that a randomly selected cedar tree will grow more than 9 inches in a given year.

g. Find the probability that a randomly selected cedar tree will grow exactly 7 inches in a given year.

9. A particular employee arrives to work sometime between 8:00 AM and 8:30 AM. Based on past experience the company has determined that the employee is equally likely to arrive at any time between 8:00 AM and 8:30 AM.

a. On average, what time does the employee arrive?

b. What is the standard deviation of the time at which the employee arrives?

c. If a call comes in for the employee at 8:10 AM find the probability that the employee will be there to take the call.

d. Find the probability that the employee will arrive between 8:20 AM and 8:25 AM.

e. Find the probability that the employee will arrive after 8:15 AM.

f. Find the probability that the employee will arrive at exactly 8:10 AM.

8.2 The Normal Distribution

Now what if the values of a random variable are not expected to be distributed evenly across a sample space? Specifically, what if the majority of values fall somewhere around the middle of the data range, with fewer values falling towards the ends of the range? The normal distribution is one example of this type of distribution.

To be sure, the normal distribution is the preeminent distribution used in the statistical theory we will examine. Many statistical inference procedures either directly or indirectly have roots in normal theory. These procedures usually assume that the population from which a random sample is drawn is normally distributed.

The **normal distribution**, also called the Gaussian distribution, was named after Carl Gauss who published a work in 1823 describing the mathematical definition of the distribution.[1] Gauss developed this distribution to describe the error in predicting the orbits of planets.

Normal distributions are all bell-shaped, but the bells come in various shapes and sizes. Since all normal distributions are symmetric, the mean, mode, and median are all equal.

Although normally distributed random variables can range in value from minus infinity to positive infinity, values that are a great distance from the mean rarely occur.

The Origins of the Normal Distribution: Abraham de Moivre (1667 – 1754)

De Moivre was born in France but lived most of his life in England. In 1733, he published a paper containing the equation that describes the normal curve. He allegedly was doing calculations using the binomial distribution for gamblers and was looking for a shortcut in the very arduous calculations. He discovered the normal distribution as the limit of the binomial distribution. De Moivre was a highly respected mathematician and friend of Issac Newton. He lived to a quite old age for the time and died in obscure poverty.

De Moivre's discovery received little attention until Laplace began writing on probability in the 1770s.

There are two other mathematicians who discovered the equation of the normal curve, Adrain in 1808 and Gauss in 1809. Even though de Moivre published the equation for the normal distribution more than 75 years earlier than Gauss, the normal curve was called the Gaussian distribution for many years. Even now, you will hear the normal curve referred to as the Gaussian distribution.