

## Variance of a Hypergeometric Random Variable

The **variance** of a hypergeometric random variable is given by the expression

$$\sigma^2 = V(X) = n \left( \frac{r}{N} \right) \left( 1 - \frac{r}{N} \right) \frac{(N-n)}{(N-1)}.$$

FORMULA

Compute the expected value and variance for the random variable defined in Example 7.6.1.

### Solution

$$E(X) = 16 \left( \frac{2}{30} \right) \approx 1.067$$

$$V(X) = 16 \left( \frac{2}{30} \right) \left( 1 - \frac{2}{30} \right) \frac{(30-16)}{(30-1)} \approx 0.481$$

Thus, if the experiment were repeated many times, the average number of defective chips per board would be slightly greater than 1.

### Example 7.6.2

**Determining the Expected Value and Variance of a Hypergeometric Random Variable**

## 7.6 Exercises

### Basic Concepts

1. How does the hypergeometric model differ from the binomial model?
2. What is the hypergeometric probability distribution function?
3. What are the parameters of the hypergeometric model?
4. How do you determine the expected value of a hypergeometric random variable? The variance?

### Exercises

5. Suppose a batch of 50 light bulbs contains 3 light bulbs that are defective. Let  $X$  = the number of defective light bulbs in a random sample of 10 light bulbs (where the sample is taken without replacement).
  - a. What probability model would be appropriate for describing the number of defective light bulbs in the sample?
  - b. Find the expected number of defective bulbs.
  - c. Find the standard deviation of the number of defective bulbs.
  - d. Find the probability that at least 1 of the bulbs sampled will be defective.
  - e. Find the probability that at most 2 of the bulbs sampled will be defective.
  - f. Find the probability that more than 3 of the bulbs sampled will be defective.

6. A small electronics firm has 60 employees. Ten of the employees are older than 55. An attorney is investigating a client's claim regarding age discrimination. The attorney randomly selects 15 employees without replacement and records the number of employees over age 55.
  - a. What probability model would be appropriate for describing the number of employees over age 55 in a sample of 15 selected without replacement?
  - b. Find the average number of employees over age 55 in the sample.
  - c. Find the standard deviation of the number of employees over age 55 in the sample.
  - d. Find the probability that at least 2 of the employees selected will be over age 55.
  - e. Find the probability that less than 2 of the employees selected will be over age 55.
  - f. Find the probability that at most 4 of the employees will be over age 55.
  
7. A bank has to repossess 100 homes. Fifty of the repossessed homes have market values that are less than the outstanding balance of the mortgage. An auditor randomly selects 10 of the repossessed homes (without replacement) and records the number of homes that have market values less than the outstanding balance of the mortgage.
  - a. Find the expected number of homes the auditor will find with market values less than the outstanding balance of the mortgage. Find the standard deviation of the number of homes the auditor will find with market values less than the outstanding balance of the mortgage.
  - b. What is the probability that all of the audited homes will have market values in excess of the outstanding balance of the mortgage?
  - c. What is the probability that none of the audited homes will have market values in excess of the outstanding balance of the mortgage?
  
8. A small liberal arts college in the Northeast has 200 freshmen. Eighty of the freshmen are engineering majors. Suppose thirty freshmen are randomly selected (without replacement).
  - a. Find the expected number of engineering majors in the sample.
  - b. Find the standard deviation of the number of engineering majors in the sample.
  - c. Find the probability that none of the selected students will be an engineering major.
  - d. Find the probability that all of the selected students will be an engineering major.