Poisson Random Variables for Length or Space

Instead of counting the number of successes in a time interval, there are a number of applications of the Poisson that measure the number of successes in some area or length. The average number of successes in the area or length will define the parameter of the Poisson random variable.

Example 7.5.2

Determining a Probability Using the Poisson Distribution

The telephone company is considering purchasing optical cable from Optica, Inc. The company wishes to replace approximately 100,000 feet of conventional cable with optical fiber. Since optical fiber is very difficult to repair, it is important that the number of optical cable defects are minimized. Optica claims that on average there is one defect per 200,000 feet of cable. What is the probability that the replaced cable will contain no defects?

Solution

Let X = the number of defects in 100,000 feet of optical cable.

Based on previous experience, we assume that the number of defects is approximated by a Poisson distribution with Poisson parameter λ computed as follows.

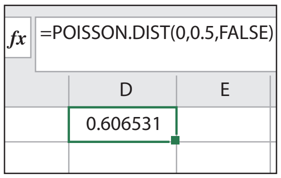
$$\lambda = \frac{100{,}000}{200{,}000} = \frac{1}{2} \quad \text{(the average number of defects per 100,000 feet of cable)}$$

Using the tables provided in Appendix A, Table F,

$$P(X = 0) = 0.6065.$$

Technology

A Poisson probability can also be found using the POISSON.DIST function in Excel. For instructions please visit stat.hawkeslearning.com and navigate to **Discovering Statistics and Data, Fourth Edition > Technology Instructions > Poisson Distribution > Poisson Probability (pdf)**.



7.5 Exercises

Basic Concepts

1. How is the Poisson distribution similar to the binomial distribution?
2. What are the two conditions that an experiment must meet in order to be considered a Poisson random variable?
3. What are some uses of the Poisson probability model in the real world?
4. What is the Poisson probability distribution function?
5. What is the parameter of the Poisson probability model?
6. What is the expected value of a Poisson random variable? The variance? The standard deviation?

Exercises

7. In the last six months, on average 5 shoppers downtown had their automobiles broken into each month while they shopped. What is the probability that exactly 2 shoppers will have their automobiles broken into next month?

8. The number of calls received by an office on Monday morning between 8:00 AM and 9:00 AM has a Poisson distribution with λ equal to 4.0.

a. Determine the probability of getting no calls between eight and nine in the morning.

b. Determine the probability of getting exactly five calls between eight and nine in the morning.

c. What will be the expected number of calls received by the office during this time period? What is the variance?

d. Graph the probability distribution of the number of calls using values from Appendix A, Table F.

9. The director of a local hospital is studying the occurrence of medication errors. Medication errors are deemed to occur when a patient is given the wrong amount of medication, or the wrong medication is given to a patient. Based on past experience, the director believes that medication errors follow a Poisson process with an average rate of 2 per week. (For the following problems, assume that 1 month = 4 weeks.)

a. What is the probability that there are no medication errors in one week?

b. What is the probability that there are no medication errors in one month?

c. Find the average number of medication errors in one week.

d. Find the average number of medication errors in one month.

e. Find the standard deviation of the number of medication errors in one month.

f. How likely is it that at least 4 medication errors will be observed in one month?

10. The number of weaving errors in a 20 ft by 10 ft roll of carpet has a Poisson distribution with $\lambda = 0.1$.

a. Using Appendix A, Table F, construct the probability distribution for the carpet.

b. What is the probability of observing less than 2 errors in the carpet?

c. What is the probability of observing more than 5 errors in the carpet?

11. A bank is evaluating their staffing policy to assure they have sufficient staff for their drive-up window during the lunch hour. If the number of people who arrive at the window in a 15-minute period has a Poisson distribution with $\lambda = 5$, answer the following questions.

a. How many people are expected to arrive during the lunch hour?

b. What is the probability that no one will show up during the lunch hour of 12:00 PM to 1:00 PM?

c. What is the probability that more than 6 people will show up in any 15-minute period?

12. An aluminum foil manufacturer wants to improve the quality of the product and is trying to develop a probability model for the flaws that occur in a sheet of foil. Assume that X, the number of flaws per square foot, has a Poisson distribution. If flaws occur randomly at an average of one flaw per 50 square feet, what is the probability that a box containing a 200 square foot roll will contain one flaw? More than one flaw?

13. A manufacturing company is concerned about the high rate of accidents that occurred on the production line last week. There were 6 accidents in the last week and this may require a report to be sent to the government agency for safety. Determine the probability of 6 accidents occurring in a week when the average number of accidents per week has been 3.5. Assume that the number of accidents per week follows a Poisson distribution.

7.6 The Hypergeometric Distribution

The binomial and the hypergeometric random variables are very similar. Both random variables have only two outcomes on each trial of the experiment. They both count the number of successes in n trials of an experiment. The hypergeometric distribution differs from the binomial distribution in the lack of independence between trials, which also implies that the probability of success will vary between trials. In addition, hypergeometric distributions have finite populations in which the total number of successes and failures are known. Hypergeometric distributions are widely used to model probability in various fields of biology: molecular biology, evolutionary biology, bioinformatics, cancer research, genomics, and malaria research.

Hypergeometric Probability Distribution

The **hypergeometric probability distribution** can be used when sampling from a population of finite size N without replacement and it is known that there are r successes in the population (therefore, $N - r$ failures). The hypergeometric distribution is used to find the probability of x successes in a sample of size n.

DEFINITION

Because the binomial and hypergeometric are closely related, a small change in an experiment can switch the distribution of the random variable. A binomial experiment, such as counting the number of red cards drawn in 8 draws from a deck with replacement, can easily be modified to a hypergeometric by not replacing the cards. Since there are 26 red cards (successes) and 26 black cards (failures), the probability of drawing a red card on the first draw is $\frac{26}{52}$ or $\frac{1}{2}$. If a red card is drawn on the first draw and not replaced, the probability of drawing a red card on the next draw is slightly less $\frac{25}{51}$, since there is one less red card in the deck. If the next card drawn is also red, then the probability of a red card on the third draw will be diminished to $\frac{24}{50}$.