Variance and Standard Deviation of a Binomial Random Variable

To find the **variance** of a binomial random variable, use the expression

$$\sigma^2 = V(X) = np(1 - p).$$

Therefore, the **standard deviation** of a binomial random variable is given by

$$\sigma = \sqrt{V(X)} = \sqrt{np(1-p)}.$$

FORMULA

Example 7.4.6

Computing the Expected Value and Variance of a Binomial Random Variable

Compute the expected value and the variance of the number of profitable leases in Example 7.4.3.

Solution

Since the random variable is binomial, we can use the formulas above. Since $n = 12$ and $p = 0.1$, the expected value is given by the following expression.

$$\begin{aligned}\mu &= E(X) = np \\ &= 12(0.1) \\ &= 1.2\end{aligned}$$

The variance is

$$\begin{aligned}\sigma^2 &= V(X) = np(1-p) \\ &= 12(0.1)(0.9) \\ &= 1.08,\end{aligned}$$

which implies that the standard deviation is $\sqrt{1.08} \approx 1.039$.

Thus, if groups of 12 oil leases were purchased with the same probability of success (0.1 probability of a profitable lease), then the average number of profitable leases per group of 12 would be 1.2 and the standard deviation would be 1.039 leases.

7.4 Exercises

Basic Concepts

1. Describe the characteristics of a binomial experiment.
2. What are the parameters of a binomial probability model?
3. Give an example of a binomial experiment, other than the one used in the section.
4. What is the formula for the binomial probability distribution function?
5. What influences the shape of the binomial probability distribution?
6. How do you determine the expected value of a binomial random variable? The variance? The standard deviation?

Exercises

7. Compute ${}_nC_x$ for each of the following combinations of x and n.

a. $n = 5, x = 4$

b. $n = 10, x = 8$

c. $n = 15, x = 1$

d. $n = 20, x = 0$

8. Compute ${}_nC_x$ for each of the following combinations of x and n.

a. $n = 4, x = 2$

b. $n = 12, x = 8$

c. $n = 18, x = 15$

d. $n = 23, x = 20$

9. The random variable X is a binomial random variable with $n = 9$ and $p = 0.1$.

a. Find the expected value of X.

b. Find the standard deviation of X.

c. Find the probability that X equals 2. (Use the formula for $P(X = x)$.)

d. Find the probability that X is at most 3.

e. Find the probability that X is at least 2.

f. Find the probability that X is less than 5.

10. The random variable X is a binomial random variable with $n = 12$ and $p = 0.8$.

a. Find the expected value of X.

b. Find the standard deviation of X.

c. Find the probability that X equals 7. (Use the formula for $P(X = x)$.)

d. Find the probability that X is at most 4.

e. Find the probability that X is at least 1.

f. Find the probability that X is more than 10.

11. A real estate agent has ten properties that she shows. She feels that there is a ten percent chance of selling any one property during a week. The chance of selling any one property is independent of selling another property.

a. What probability model would be appropriate for describing the number of properties sold each week?

b. Compute the expected number of properties to be sold in a week.

c. Compute the standard deviation of the number of properties sold each week.

d. Compute the probability of selling one property in one week.

e. Compute the probability of selling five properties in one week.

f. Compute the probability of selling at least three properties in one week.

12. A small commuter airline is concerned about reservation no-shows and, correspondingly, how much they should overbook flights to compensate. Assume their commuter planes will hold 15 people. Industry research indicates that 20% of the people making a reservation will not show up for a flight. Whether or not one person takes the flight is considered to be independent of other persons holding reservations.

a. What probability model would be appropriate for the number of passengers that actually take the flight?

b. If the airline decides to book 18 people for each flight, how often will there be at least one person who will not get a seat?

c. If they book 17 people, how often will there be at least one person who will not get a seat?

d. If they book 16 people, how often will there be at least one person who will not get a seat?

e. If they book 18 people for each flight, how often will there be one or more empty seats?

f. If they book 17 people, how often will there be one or more empty seats?

g. If they book 16 people, how often will there be one or more empty seats?

h. Based on the results from parts **b.** to **g.** above, which booking policy do you prefer? Explain your answer.

13. Seven plants are operated by a garment manufacturer. They feel there is a ten percent chance for a power outage to occur in a month at any one plant and the risk of a power outage at one plant is independent of the risk of a power outage at another plant. Let X = the number of plants of the garment manufacturer that have a power outage in the next month.

a. Determine the probability distribution for X.

b. Interpret the results for $P(X=0)$, $P(X=4)$, and $P(X=7)$.

c. Compute the expected value of X.

d. Compute the standard deviation for X. Is this value large in relation to the expected value? In what units is the standard deviation expressed?

14. A company that makes traffic signal lights buys switches from a supplier. Out of each shipment of 1000 switches, the company will take a random sample of 10 switches. Let X equal the number of defective switches in the sample.

a. The company has a policy of rejecting a lot if they find any defective switches in the sample. What is the probability that the shipment will be accepted if, in fact, 2% of the switches are actually defective?

b. What is the probability that the shipment will be accepted if the percent of defective switches is actually 5%?

c. The company decides to change their policy and will accept the lot if they find no more than one defective switch. Repeat parts a. and b. for this new policy.

15. The probability of getting into medical school if one or both of your parents is a doctor is 0.7. If a doctor has four children answer the following questions.

a. Determine the probability of getting exactly two children into medical school.

b. What is the probability of getting at least two children into medical school?

16. A certain aspirin is advertised as being preferred by 4 out of 5 doctors. If the advertisement is assumed to be true, answer the following questions.

a. What is the probability that at least half of ten doctors chosen at random will prefer this brand of aspirin?

b. What is the probability that 9 out of 10 of the doctors will prefer this brand?

17. In manufacturing integrated circuits, the yield of the manufacturing process is the percentage of good chips produced by the process. The probability that an integrated circuit manufactured by the Ace Electronics Company will be defective is $p = 0.05$. If a random sample of 15 circuits is selected for testing, answer the following questions.

a. What is the probability that no more than one integrated circuit will be defective in the sample?

b. What is the expected number of defective integrated circuits in the sample?

18. A local employment service procures temporary office personnel for local businesses. They have found that 90% of the invoices for their services are paid within 10 working days. If a random sample of 12 invoices is checked, answer the following questions.

a. What is the probability that all of the invoices will be paid within 10 working days?

b. What is the probability that six or more of the invoices will be paid within 10 working days?

19. An experiment consists of rolling a pair of typical six-sided dice 10 times. On each roll the sum of the dots on the two dice is noted.

a. Find the probability that on any roll of the two dice the sum of the dots is either 7 or 11.

b. Find the probability that in the 10 rolls of the pair of dice, a 7 or 11 occurs 5 times.

c. Find the probability that in the 10 rolls of the pair of dice, a 7 or 11 does not occur at all.

d. Find the mean and variance of the number of times we see a 7 or 11 in the 10 rolls of the dice.

20. "Would you say you eat to live or live to eat?" was asked to each person in a sample of 1001 adults in a recent survey. Seventy-four percent of the respondents answered eat to live, 23% answered live to eat, and 3% had no opinion. Assuming these percentages are accurate, find the probability, in 12 randomly chosen adults, that the number who would answer "eat to live" is:

a. exactly 7.

b. no more than 10.

c. at most 11.

d. at least 3.