

## 7.2 Exercises

### Basic Concepts

1. Why is the notion of expected value one of the most important concepts in the analysis of random phenomena?
2. True or False: The expected value of a discrete random variable is usually one of the possible outcomes of the random variable.
3. Suppose the expected value of a random variable was known to be 6.3. Interpret the meaning of the expected value.
4. Give an example of a situation in which expected value would be useful to compare alternatives, other than the one used in the section.
5. How is the variability of a random variable related to risk?
6. True or False: Two random variables that have exactly the same probability distributions will have the same expected value.
7. True or False: Two random variables with the same expected value will have the same probability distributions.

### Exercises

8. Determine the expected value, the variance, and the standard deviation for a random variable with the following probability distribution.

$x$	-5	-2	0	2	5
$p(x)$	0.06	0.15	0.58	0.18	0.03

9. Determine the expected value, the variance, and the standard deviation for a random variable with the following probability distribution.

$x$	400	420	440	460	480	500
$p(x)$	0	0.1	0.1	0.2	0.2	0.4

10. A regional hospital is considering the purchase of a helicopter to transport critical patients. The relative frequency of  $X$ , the number of times the helicopter is used to transport critical patients each month, is derived for a similarly sized hospital and is given in the following probability distribution.

$x$	0	1	2	3	4	5	6
$p(x)$	0.15	0.20	0.34	0.19	0.06	0.05	0.01

- a. Determine the average number of times the helicopter is used to transport critical patients each month.
- b. Determine the variance of the number of times the helicopter is used to transport critical patients.
- c. Determine the standard deviation of the number of times the helicopter is used to transport critical patients.

- d. Determine the probability that the helicopter will not be used at all during a month to transport critical patients.
- e. Determine the probability that the helicopter will be used at least once to transport critical patients.
- f. Determine the probability that the helicopter will be used at most twice to transport critical patients.
- g. Determine the probability that the helicopter will be used more than three times to transport critical patients.
11. Based on past experience, an architect has determined a probability distribution for  $X$ , the number of times a drawing must be examined by a client before it is accepted.

$x$	1	2	3	4	5
$p(x)$	0.1	0.2	0.3	0.2	0.2

- a. Find the average number of times a drawing must be examined by a client before it is accepted.
- b. Find the variance of the number of times a drawing must be examined by a client before it is accepted.
- c. Find the standard deviation of the number of times a drawing must be examined by a client before it is accepted.
- d. What is the probability that a drawing must be examined five times before being accepted by the client?
- e. Find the probability that the drawing must be examined at least twice before being accepted by the client.
- f. Find the probability that a drawing must be examined at most three times before being accepted by the client.
- g. Find the probability that a drawing must be examined less than twice before being accepted by the client.
12. The manager of a retail clothing store has determined the following probability distribution for  $X$ , the number of customers who will enter the store on Saturday.

$x$	10	20	30	40	50	60
$p(x)$	0.10	0.20	0.30	0.20	0.10	0.10

- a. Find the expected number of customers who will enter the store on Saturday.
- b. Find the standard deviation of the number of customers who will enter the store on Saturday.
- c. Find the variance of the number of customers who will enter the store on Saturday.
- d. Find the probability that more than 30 customers will enter the store on Saturday.
- e. Find the probability that at most 20 customers will enter the store on Saturday.
- f. Find the probability that at least 40 customers will enter the store on Saturday.
- g. What is the probability that exactly 10 customers will enter the store on Saturday?

13. An entrepreneur is considering investing in a new venture. If the venture is successful, he will make \$50,000. However, if the venture is not successful, he will lose his investment of \$10,000. Based on past experience, he believes that there is a 40% chance that the venture will be successful.
- Use the information in the problem to determine the probability distribution of the amount of money to be made (or lost) on the venture.
  - Determine the expected amount of money to be made on the venture.
  - Determine the standard deviation of the amount of money to be made on the venture.
14. An investor is considering two alternative investment options with the following payoff distributions.

	Option 1			Option 2		
Payoff	-\$100,000	\$30,000	\$100,000	-\$20,000	\$0	\$20,000
$P(\text{Payoff})$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0.25	0.50	0.25

- Determine the expected payoff for each of the investment options.
  - Determine the standard deviation of the payoff for each of the investment options.
  - Which investment option would you choose? Explain.
15. A cereal manufacturer has two new brands of cereal which it would like to produce. Because resources are limited, the cereal manufacturer can only afford to produce one of the new brands. A marketing study produced the following probability distributions for the amount of sales for each of the new brands of cereal.

Cereal A		Cereal B	
Sales	$P(\text{Sales})$	Sales	$P(\text{Sales})$
\$150,000	0.2	\$10,000	0.40
\$200,000	0.3	\$300,000	0.40
\$300,000	0.3	\$600,000	0.10
\$400,000	0.2	\$1,000,000	0.10

- What are the expected sales of each of the new brands of cereal?
- What is the standard deviation of the sales for each of the brands of cereal?
- If both of the brands of cereal cost the same amount to produce, which brand of cereal do you think the cereal manufacturer should produce? Explain.