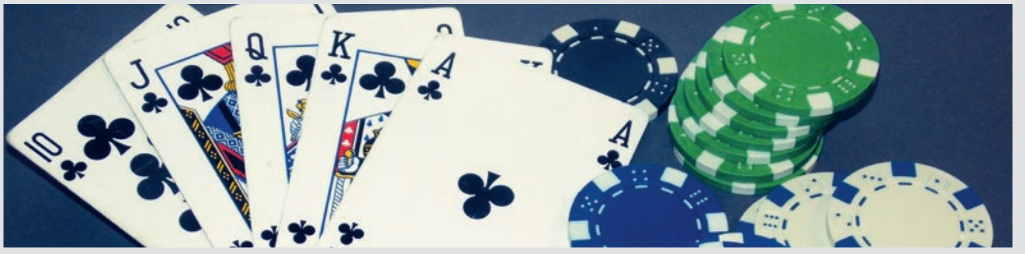
$$P(B \mid A) = \frac{\begin{pmatrix}\text{number of non-spades in the unknown cards remaining} \\ \text{in the deck given the } 4^{\text{th}} \text{community card was a non-spade}\end{pmatrix}}{\text{total number of unknown cards remaining in the deck}} = \frac{36}{44} \approx 0.818182.$$

Therefore,

$$P(\text{neither CC is a spade}) = P(A \cap B) = P(A)\ P(B|A) \approx 0.822222 \cdot 0.818182 \approx 0.6727.$$

There is approximately a 67.27% chance of getting non-spades in the next two CCs. Would you call her bet?



Texas Hold'em

Here is a brief overview of how the game is played.

1. The game begins with each player being dealt two cards face down, which are called "hole cards" or "pocket cards."
2. The first round of betting begins with the player to the left of the dealer, who has the option to "call" (match the current bet), "raise" (increase the current bet), or "fold" (discard their hand and end their participation in the current hand). All of the bets throughout the rounds comprise the "pot."
3. After the first round of betting, three community cards are dealt face up in the center of the table, which are called the "flop." These cards can be used by all players to make their best five-card hand.
4. A second round of betting takes place, beginning with the player to the left of the dealer.
5. A fourth community card is dealt face up in the center of the table, which is called the "turn."
6. A third round of betting takes place, beginning with the player to the left of the dealer.
7. A fifth and final community card is dealt face up in the center of the table, which is called the "river."
8. A final round of betting takes place, beginning with the player to the left of the dealer.
9. If more than one player is still in the hand after the final betting round, the players reveal their hole cards and the player with the best five-card hand wins the "pot."

6.3 Exercises

Basic Concepts

1. Define conditional probability.
2. How do you calculate the conditional probability $P(A|B)$?
3. Explain the difference between dependent and independent events.
4. Are mutually exclusive events dependent or independent? Explain your answer.

5. If events *A* and *B* are independent, what is $P(A|B)$ equal to?

6. What is the product rule?

7. In the case *People v. Collins* an appeals court overturned the conviction. What flaws did the appeals court detect in the case against the accused assailants?

8. What does it mean to sample with replacement?

Exercises

9. A health care provider classifies its customers by their housing situation and whether they have health insurance coverage. The market research department has gathered data from a random sample of 759 customers.

Health Care Consumers

Have Health Insurance Coverage	Housing Situation	
	Rent	Own
Yes	196	298
No	92	173

a. Given that the customer rents their home, what is the probability that the customer does not have health insurance?

b. Given that the customer does not have health insurance, what is the probability that the customer rents their home?

c. Given that the customer owns their home, what is the probability that the customer has health insurance?

d. Given that the customer has health insurance, what is the probability that the customer owns their home?

10. The following table was given in Section 6.2, Exercise 15.

Life Insurance Coverage

		Amount of Life Insurance on Husband ($)			
		0–249,999	250,000–499,999	500,000–999,999	1,000,000 or more
Amount of Life Insurance on Wife ($)	0–249,999	400	200	50	50
	250,000–499,999	50	50	30	30
	500,000–999,999	20	10	25	25
	1,000,000 or more	20	10	15	15

a. Given the wife has between $500,000 and $999,999 of insurance, what is the probability that the husband has $1,000,000 or more of insurance?

b. Given the wife has between $0 and $249,999 of insurance, what is the probability that the husband has between $0 and $999,999 of insurance?

c. Given that the husband has between $0 and $250,000 of insurance, what is the probability that the wife will have $1,000,000 or more of insurance?

d. Given that the husband has $1,000,000 or more of insurance, what is the probability that the wife will have $1,000,000 or more of insurance?

11. A computer software company receives hundreds of support calls each day. There are several common installation problems, call them A, B, C, and D. Several of these problems result in the same symptom, *lock up* after initiation. Suppose that the probability of a caller reporting the symptom *lock up* is 0.7 and the probability of a caller having problem A and a *lock up* is 0.6.

    a. Given that the caller reports a lock up, what is the probability that the cause is problem A?

    b. What is the probability that the cause of the malfunction is not problem A given that the caller is experiencing a lock up?

12. A television advertising representative has determined the following probabilities based on past experience. The probability that an individual will watch an ad during the Super Bowl is 0.10. Given that the individual watches the ad, the probability that the individual will buy the product is 0.005. It is also known that the probability that an individual would buy the product is 0.02. Given that an individual buys the product, find the probability that the individual watched the television ad during the Super Bowl.

13. Medical researchers have determined that there is a 2% chance that an individual will have a gene which gives him a predisposition for heart disease. Given that an individual has the gene, the probability that heart disease will develop is 25%. It is also known that the probability that an individual has heart disease is 12%.

    a. Find the probability that an individual will have the gene and develop heart disease.

    b. Given that a person has heart disease, what is the probability that they have the gene?

14. Use the table given in Exercise 9.

    Are the events {customer rents their home} and {customer owns their home} independent? Explain.

15. Use the table given in Exercise 10.

    Are the events {the husband has $1,000,000 or more in insurance} and {the wife has $250,000 or more in insurance} independent? Explain.

16. Suppose you were flipping a coin. What is the probability that you would observe a head:

    a. on two consecutive flips?

    b. on three consecutive flips?

    c. on four consecutive flips?

    d. on 100 consecutive flips?

17. Suppose an atomic reactor has two independent cooling systems. The probability that Cooling System A will fail is 0.01 and the probability that Cooling System B will fail is 0.01. What is the probability that both systems will fail simultaneously?

18. Mandy is 30, and the probability that she will survive until age 65 is 0.90. Ashley is 45, and the probability that she will survive until age 65 is 0.95.

a. Find the probability that both Mandy and Ashley will survive until age 65.

b. Find the probability that only Mandy will survive until age 65.

c. Find the probability that neither Mandy nor Ashley will survive until age 65.

d. What assumption about the lives of Mandy and Ashley did you make in answering the above questions?

19. An insurance company is considering insuring two large oil tankers against spills. The limit of the liability on the coverage is $10,000,000. The company believes that the probability of an oil spill requiring the maximum liability coverage during the policy period is 0.001 per tanker.

a. What is the probability that neither tanker would have a spill requiring the maximum liability coverage during the policy period?

b. What is the probability that only one tanker would have a spill requiring the maximum liability coverage during the policy period?

c. What is the probability that both tankers would have spills requiring the maximum liability coverage during the policy period?

20. Coin flipping can be used to model other real life phenomena and aid in certain probability calculations. An example of this would be to compute the probability that the World Series ends in some specified number of games. The World Series is a best of seven game series played at the end of the regular baseball season between the champion of the American League and the champion of the National League. The first team to win four games is declared the champion of baseball for that year. If we assume the probability of either team winning a game is approximately 0.5 and the games are independent events, the probability that the series ends in either 4, 5, 6, or 7 games can be computed.

a. What is the probability that the series ends in exactly 4 games? Write the sample space consisting of 16 equally likely outcomes similar to the sample space resulting from tossing a coin four times.

b. What is the probability that the series ends in exactly 5 games?

c. Assume the probability that the series ends in exactly 6 games is $\frac{5}{16}$. Use this information together with your answers to the first two parts of this problem to compute the probability that the series ends in exactly 7 games.

21. Drug usage in the workplace costs employers incredible amounts of money each year. Drug testing potential employees has become so prevalent that drug users are finding it extremely hard to find jobs. Drug tests, however, are not completely reliable. The most common test used to detect drugs is approximately 98% accurate. To decrease the likelihood of making an error, all potential employees are screened through two tests, which are independent, and each has about 98% accuracy.

a. If a person were drug-free, what is the probability they would fail both tests?

b. If a person were a drug user, what is the probability they would pass both tests?

22. Suppose you draw two cards out of a standard deck without replacement. What is the probability that you draw the ace of spades and then another spade?

23. Manchester United F.C. has 42 players on their first team roster, classified by position and age group as follows.

Manchester United F.C. First Team Roster

	18-22 year olds	23-30 year olds
Defender	9	13
Midfielder	4	5
Forward	4	2
Goalkeeper	2	3

Their head coach must choose two players at random for a press conference.

a. What is the probability that a midfielder age 18-22 and a defender age 23-30 are chosen?

b. What is the probability that two players age 18-22 are chosen?

6.4 Combinations and Permutations

To compute certain probabilities, such as the probability of having winning numbers in the state lottery, requires the ability to count the number of possible outcomes for a given experiment or a sequence of experiments.

However, often it is impractical to list out all the possibilities. Therefore, we will develop some techniques to facilitate counting.

The Fundamental Counting Principle

E_1 is an event with n_1 possible outcomes and E_2 is an event with n_2 possible outcomes. The number of ways the events can occur in sequence is $n_1 \cdot n_2$. This principle can be applied for any number of events occurring in sequence.

PROCEDURE

Example 6.4.1

Using the Fundamental Counting Principle to Count the Number of Tomato Plots Needed for an Experiment

An agricultural research center is designing an experiment to determine the effects of soil type, fertilizer, and plant spacing on tomato yield. The plan calls for four soil types, three types of fertilizer, and four different spacings between plants. The plan also includes replication of each combination of fertilizer, soil type, and plant spacing five times. How many different plots of tomatoes will be needed to conduct the experiment?

Solution

$$\underset{\text{(soil types)}}{4} \cdot \underset{\text{(fertilizers)}}{3} \cdot \underset{\text{(plant spacings)}}{4} \cdot \underset{\text{(replications)}}{5} = 240$$