

6.2 Exercises

Basic Concepts

1. What laws must probability obey, regardless of the methodology used to derive the probabilities?
2. Suppose you are taking a test next week. Interpret each of the following statements.
 - a. $P(\text{receiving an A on the test}) = 0$
 - b. $P(\text{receiving an A on the test}) = 1$
3. What is a compound event?
4. Draw a Venn diagram to represent the intersection of three events.
5. Define the following set operations: union, intersection, and complement.
6. If you know the probability of two events, what else must you know in order to determine the probability of *one event or the other*?
7. If two events A and B are mutually exclusive, what is $P(A \cap B)$?

Exercises

8. Determine if the following values could be probabilities. If the value cannot be a probability, explain why.

a. 0	d. -0.4
b. $\frac{36}{25}$	e. 0.23
c. $\frac{7}{8}$	
9. Determine if the following values could be probabilities. If the value cannot be a probability, explain why.

a. 1	d. 0.99
b. $\frac{15}{16}$	e. -0.05
c. $\frac{4}{3}$	
10. Interpret the following probabilities with respect to the occurrence of some event.

a. $P(\text{event}) = 0$	d. $P(\text{event}) = 65\%$
b. $P(\text{event}) = 1.0$	e. $P(\text{event}) = -1.0$
c. $P(\text{event}) = 0.45$	
11. Find the following probabilities.
 - a. The probability of an event that must happen.
 - b. The probability of an event that cannot happen.
 - c. The probability of rolling an even number in a single toss of a six-sided die.
 - d. The probability of rolling a two and a five in a single toss of a six-sided die.

12. Find the following probabilities related to odds.
- If the odds in favor of an event A occurring is 3:5, what is the probability of event A?
 - If the odds against an event A occurring is 3:5, what is the probability of event A?
13. The annual premium amounts charged by life insurance companies to their clients are set very carefully. If the amount is too high, the client will take his or her business to another company. If it is too low, the insurance company may not make enough profit to stay in business. In order to properly determine a premium, the company often relies on life tables. These tables allow one to compute the probabilities of death at various ages. They are constructed only after collecting and reviewing extensive data on age at death from a large group of people. A life table is normally constructed assuming that 100,000 people are alive at age 0. This number is simply a reference value used to make comparisons throughout the table. Other numbers could be used. The table then gives the number of people of the original 100,000 that are alive at the beginning of various years of life. In order for the insurance company to optimally set premiums, a separate table should be constructed for the different genders and races. The following abbreviated life table is valid only for females.

Life Table						
Year	0	1	5	10	15	20
Number Alive	100,000	99,090	98,912	98,815	98,716	98,477
Year	25	30	35	40	45	50
Number Alive	98,204	97,897	97,500	96,958	96,097	94,766
Year	55	60	65	70	75	80
Number Alive	92,623	89,449	84,565	77,772	68,200	55,535

- What is the probability that a newborn female lives until the age of 40?
 - What is the probability that a newborn female dies before she reaches the age of 50?
14. A health care provider classifies its customers by their housing situation and whether they have health insurance coverage. The market research department has gathered data from a random sample of 759 customers.

Health Care Consumers		
Have Health Insurance Coverage	Housing Situation	
	Rent	Own
Yes	196	298
No	92	173

- What is the probability that a customer rents their home?
- What is the probability that a customer owns their home?
- What is the probability that a customer has health insurance coverage and rents their home?

- d. What is the probability that a customer owns their home and does not have health insurance coverage?
- e. What is the probability that a customer has health insurance coverage and rents their home or does not have health insurance coverage and owns their home?
- f. What is the probability that a customer does not have health insurance coverage?
- g. What approach to probability did you use to calculate your answers?
- h. Are the events {rents their home} and {owns their home} mutually exclusive? Explain.

15. A large life insurance company is interested in studying the insurance policies held by married couples. In particular, the insurance company is interested in the amount of insurance held by the husbands and the wives. The insurance company collects data for all of its 1000 policies where both the husband and the wife are insured. The results are summarized in the following table.

Life Insurance Coverage					
		Amount of Life Insurance on Husband (\$)			
		0-249,999	250,000-499,999	500,000-999,999	1,000,000 or more
Amount of Life Insurance on Wife (\$)	0-249,999	400	200	50	50
	250,000-499,999	50	50	30	30
	500,000-999,999	20	10	25	25
	1,000,000 or more	20	10	15	15

- a. For a randomly selected policy, what is the probability that the husband will have between \$250,000 and \$499,999 of insurance?
- b. For a randomly selected policy, what is the probability that the wife will have between \$500,000 and \$999,999 of insurance?
- c. For a randomly selected policy, what is the probability that the wife will have \$1,000,000 or more of insurance or the husband will have \$1,000,000 or more of insurance?
- d. For a randomly selected policy, what is the probability that the wife will have between \$0 and \$249,999 of insurance and the husband will have between \$0 and \$249,999 of insurance?
- e. For a randomly selected policy, what is the probability that the wife will not have between \$0 and \$249,999 of insurance?
- f. For a randomly selected policy, what is the probability that the husband will have \$250,000 or more of insurance?
- g. What approach to probability did you use to determine your answers?
- h. Are the events {the wife has \$1,000,000 or more in insurance} and {the husband has between \$250,000 and \$499,999 of insurance} mutually exclusive? Explain.