Test Procedure for the Wilcoxon Signed-Rank Test

Assumptions:

1. Pairs of data have been selected in a random fashion.
2. Data are quantitative.

Hypotheses:

H_0: The probability distributions of the two populations of interest are the same.

H_a: $>$ One-Tailed: Population X is to the *right* of Population Y (Diff = $Y - X$).

$\neq$ Two-Tailed: Population X is to the *right* of or to the *left* of Population Y.

$<$ One-Tailed: Population X is to the *left* of Population Y (Diff = $Y - X$).

Test Statistic:

If $n \leq 25$, and

H_a: $>$, then $T = T_+ =$ the sum of the ranks of the positive differences.

H_a: $\neq$, then $T = \text{Min}\,(T_+, T_-)$.

H_a: $<$, then $T = T_- =$ the sum of the ranks of the negative differences.

If $n > 25$, $T = \text{Min}\,(T_+, T_-)$, and the test statistic is given by

$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}.$$

Critical Value(s):

If $n \leq 25$, reject H_0 if $T \leq T_c$, the critical value in Table J.

If $n > 25$, and

H_a: $>$ or H_a: $<$, reject H_0 if $z \leq -z_\alpha$.

H_a: $\neq$, reject H_0 if $z \leq -z_{\alpha/2}$.

PROCEDURE

17.2 Exercises

Basic Concepts

1. What assumptions are required for the Wilcoxon Signed-Rank Test?

2. The Wilcoxon Signed-Rank Test is primarily used to perform hypothesis tests about what type of experiment?

3. What are the advantages and disadvantages of the Wilcoxon Signed-Rank Test?

4. Describe the procedure for assigning ranks to data in order to perform a Wilcoxon Signed-Rank Test. What is to be done when two values are the same?

5. Describe how to calculate the rank sums for a paired difference experiment in order to perform a Wilcoxon Signed-Rank Test.
6. If the sample size is less than or equal to 25, identify the three possible test statistics used for the Wilcoxon Signed-Rank Test. How do you choose which statistic to use?
7. What are the null and alternative hypotheses associated with the Wilcoxon Signed-Rank Test?
8. Explain why the population distributions are important when performing a Wilcoxon Signed-Rank Test.
9. What is the test statistic for the Wilcoxon Signed-Rank Test if the sample size is large? How large is *large* with regard to sample size?
10. Identify the critical values and rejection regions for both large and small samples with regard to the Wilcoxon Signed-Rank Test.

Exercises

11. Rank the following emerging markets mutual funds from lowest to highest price using the methodology presented for the Wilcoxon Signed-Rank Test.

Emerging Markets Mutual Funds

Mutual Fund	Price ($)
American Funds	49.30
Columbia Management	9.41
Morgan Stanley	88.50
Fidelity Investments	24.40
John Hancock	9.41
DWS Investments	15.57
UBS	12.15
Prudential Investments	9.23
Value Line Funds	32.82
The Vanguard Group	34.72

12. Rank the following consumer price indexes (CPI) for selected groups of goods and services in September 2011 using the methodology presented for the Signed-Rank Test. The data in the table represent the unadjusted percent change in price level from September 2010 to September 2011.[1]

Percent Change in CPI

Expenditure Category	CPI (% Change 9/10 to 9/11)
Food	4.7
Alcoholic Beverages	1.4
Housing	1.8
Apparel	3.5
Public Transportation	7.4
Medical Care	2.8

Percent Change in CPI	
Expenditure Category	**CPI (% Change 9/10 to 9/11)**
Education	4.4
Tobacco and Smoking Products	2.4
Gasoline	33.3
New and Used Motor Vehicles	3.6

13. A study conducted by the Orentreich Foundation found that women who practiced transcendental meditation (T.M.) for 20 minutes a day had high levels of DHEA-S, a hormone that may help prevent breast cancer and osteoporosis. Suppose eight women are randomly selected to participate in a study. The DHEA-S levels of the participants are measured prior to practicing transcendental meditation and then measured one year after practicing transcendental meditation for 20 minutes a day. The following table is a summary of the results of the study.

Study Results		
Study Participant	**DHEA-S Level Before T.M. (mg)**	**DHEA-S Level After T.M. (mg)**
A	20	25
B	25	25
C	18	20
D	27	26
E	19	20
F	24	26
G	20	21
H	30	29

a. Using the Sign Test, does the data indicate that the DHEA-S level of women increases after practicing transcendental meditation for 20 minutes per day for one year at $\alpha = 0.05$?

b. What assumptions were necessary to perform the Sign Test?

c. Using the Signed-Rank Test, does the data indicate that the DHEA-S level of women increases after practicing transcendental meditation for 20 minutes per day for one year at $\alpha = 0.05$?

d. What assumptions were necessary to perform the Signed-Rank Test?

e. Which test do you think produces more accurate results? Why?

14. The management for a large grocery store chain would like to determine if a new scanner will enable cashiers to process a larger number of items on average than the scanner which they are currently using. Seven cashiers are randomly selected, and the number of grocery items which they can process in three minutes is measured for both the old scanner and the new scanner. The results of the test are as follows.

Number of Grocery Items Processed in Three Minutes							
Cashier	1	2	3	4	5	6	7
Old scanner	60	70	55	75	62	52	58
New scanner	65	71	55	75	65	57	57

a. What assumption must be made in order to perform the test of hypothesis using the paired difference t-test?

b. Using the Signed-Rank Test, does the data provide conclusive evidence that the new scanner enables cashiers to process a significantly larger number of items than the old scanner at $\alpha = 0.05$?

c. What assumptions were made in performing the Signed-Rank Test?

d. How do the results of the Signed-Rank Test compare with the paired difference t-test performed in Section 12.3, Exercise 9?

15. An auto dealer is marketing two different models of a high-end sedan. Since customers are particularly interested in the safety features of the sedans, the dealer would like to determine if there is a difference in the braking distance (the number of feet required to go from 60 mph to 0 mph) of the two sedans. Six drivers are randomly selected and asked to participate in a test to measure the braking distance for both models. Each driver is asked to drive both models and brake once they have reached exactly 60 mph. The distance required to come to a complete halt is then measured in feet. The results of the test are as follows.

Braking Distance of High-End Sedans (in Feet)						
Driver	1	2	3	4	5	6
Model A	150	145	160	155	152	153
Model B	152	146	160	157	154	155

a. What assumption must be made in order to perform a test of hypothesis using the paired difference t-test?

b. Using the Signed-Rank Test, does the data provide conclusive evidence that there is a significant difference in the median braking distance of the two sedans at $\alpha = 0.10$?

c. What assumptions were made in performing the Signed-Rank Test?

d. How do the results of the Sign Test performed in Section 17.1, Exercise 21 and the signed-rank test performed in part **b.** compare with the paired difference t-test performed in Section 12.3, Exercise 10?

17.3 The Wilcoxon Rank-Sum Test

We discussed nonparametric procedures for testing claims about a paired difference experiment in the previous two sections. In this section we will discuss a nonparametric procedure for hypothesis tests in which an independent experimental design is used to compare two population medians.