15.3 Exercises

Basic Concepts

1. What is a completely randomized design? Give an example.
2. What are blocks? What is their purpose?
3. What is a randomized block design? How is it different from a completely randomized design?
4. What are the null and alternative hypotheses when comparing means using a randomized block design?
5. What is the breakdown of the sum of squares for a randomized block design? Does this breakdown make sense? Explain.
6. If blocking is successful, how does the value of SSE change?
7. What are the assumptions when performing a two-way ANOVA for a randomized block design?
8. What is the rationale for the test statistic used for the randomized block design?

Exercises

9. A car dealer is interested in comparing the average gas mileages of four different car models. The dealer believes that the average gas mileage of a particular car will vary depending on the person who is driving the car due to different driving styles. Because of this, he decides to use a randomized block design. He randomly selects six drivers and asks them to drive each of the cars. He then determines the average gas mileage for each car and each driver. The results of the study are as follows.

Gas Mileage (MPG)				
	Car A	**Car B**	**Car C**	**Car D**
Driver 1	33	29	27	37
Driver 2	36	32	30	40
Driver 3	34	30	28	38
Driver 4	31	27	25	35
Driver 5	33	29	27	37
Driver 6	35	33	31	41

   a. Identify the dependent variable, the treatment variable, and the blocking variable.

   b. Do you think a randomized block design is appropriate for the car dealer's study? Explain.

   c. The results of the two-way ANOVA for the dealer's survey of the average gas mileages of the different car models are given in the following table. Can the dealer conclude that there is a significant difference in average gas mileages of the four car models? Use $\alpha = 0.05$.

ANOVA

Source of Variation	*SS*	*df*	*MS*
Rows	84.8333	5	16.9667
Columns	348.5000	3	116.1667
Error	2.5000	15	0.1667
Total	435.8333	23	

d. Was the dealer able to significantly reduce variation among the observed gas mileages by blocking? Use $\alpha = 0.05$.

10. A banana grower has three fertilizers from which to choose. He would like to determine which fertilizer produces banana trees with the largest yield (measured in pounds of bananas produced). The banana grower has noticed that there is a difference in the average yields of the banana trees depending on which side of the farm they are planted (South Side, North Side, West Side, or East Side). Because of the variation in yields among the areas on the farm, he has decided to randomly select three trees within each area and then randomly assign the fertilizers to the trees. After harvesting the bananas, he calculates the yields of the trees within each of the areas. The results are as follows.

Banana Yields (Pounds)			
	Fertilizer A	**Fertilizer B**	**Fertilizer C**
South Side	53	51	58
North Side	48	47	53
West Side	50	48	56
East Side	50	47	54

a. Identify the dependent variable, the treatment variable, and the blocking variable.

b. Do you think a randomized block design is appropriate for the banana grower's study? Explain.

c. The results of the two-way ANOVA for the banana grower's study are given in the following table. Can the banana grower conclude that there is a significant difference among the average yields of the banana trees for the three fertilizers? Use $\alpha = 0.10$.

ANOVA

Source of Variation	*SS*	*df*	*MS*
Rows	36.2500	3	12.0833
Columns	104.0000	2	52.0000
Error	2.0000	6	0.3333
Total	142.2500	11	

d. Was the banana grower able to significantly reduce variation among the observed yields by blocking? Use $\alpha = 0.10$.

11. The FAA is interested in knowing if there is a difference in the average numbers of on-time arrivals for four of the major airlines. The FAA believes that the number of on-time arrivals varies by airport. To control for this variation, they randomly select 100 flights for each of the major airlines at each of four randomly selected airports and record the number of on-time flights. The results of the study are as follows.

On-Time Flights				
	Airline A	**Airline B**	**Airline C**	**Airline D**
Airport A	87	82	79	81
Airport B	88	84	81	82
Airport C	89	84	83	82
Airport D	90	86	85	83

a. Identify the dependent variable, the treatment variable, and the blocking variable.

b. Do you think a randomized block design is appropriate for the FAA's study? Explain.

c. The results of the two-way ANOVA for the FAA's study are given in the following table. Can the FAA conclude that there is a significant difference among the average number of on-time arrivals for the four major airlines? Use $\alpha = 0.01$.

ANOVA

Source of Variation	*SS*	*df*	*MS*
Rows	29.2500	3	9.7500
Columns	112.7500	3	37.5833
Error	5.7500	9	0.6389
Total	147.7500	15	

d. Was the FAA able to significantly reduce variation among the observed number of on-time arrivals by blocking? Use $\alpha = 0.01$.

12. A psychologist is interested in determining if there is a difference in the average numbers of patients for several age groups. The psychologist believes that there may be some variation in the numbers of patients depending on the region of the country (Northeast, Northwest, Southeast, or Southwest). The psychologist randomly selects 100,000 individuals from each region of the country for each of the age groups of interest and determines the number of psychology patients. The results of the study are as follows.

Psychology Patients							
	Age 15–24	**Age 25–34**	**Age 35–44**	**Age 45–54**	**Age 55–64**	**Age 65–74**	**Age 75–84**
Northeast	15	17	16	17	55	22	27
Northwest	13	16	16	16	49	19	26
Southeast	12	14	15	15	47	17	24
Southwest	13	15	15	16	53	20	25

a. Identify the dependent variable, the treatment variable, and the blocking variable.

b. Do you think a randomized block design is appropriate for the psychologist's study? Explain.

c. The results of the two-way ANOVA for the psychologist's study are given in the following table. Can the psychologist conclude that there is a significant difference among the average number of patients for the different age groups? Use $\alpha = 0.10$.

ANOVA

Source of Variation	*SS*	*df*	*MS*
Rows	44.9643	3	14.9881
Columns	4223.3571	6	703.8929
Error	25.7857	18	1.4325
Total	4294.1071	27	

d. Was the psychologist able to significantly reduce variation among the observed number of psychology patients by blocking? Use $\alpha = 0.05$.

13. In an experiment designed to compare automated blood pressure devices with those of the standard cuff method, each person in a sample of six patients has their systolic blood pressure determined by three different automated devices and by the standard cuff method. The data is given in the following table.

Blood Pressure (mm Hg)				
	Device 1	**Device 2**	**Device 3**	**Standard Cuff**
Patient 1	126	128	132	131
Patient 2	134	138	137	140
Patient 3	145	144	150	152
Patient 4	129	134	132	136
Patient 5	154	160	162	160
Patient 6	144	144	148	145

a. Identify the dependent variable, the treatment variable, and the blocking variable.

b. Why was a randomized block design used in this experiment?

c. From the data, SST and SSE were computed to be 106.4583 and 53.2917, respectively. With $\alpha = 0.05$, can we conclude that the four different methods of determining systolic blood pressure have different mean readings?

d. SSBL was computed to be 2412.8750. With $\alpha = 0.05$, can we conclude that using people as blocks significantly reduced variation in this study?

15.4 Two-Way ANOVA: The Factorial Design

The randomized block test presented in the previous section is one example of a **two-way ANOVA**. There were two independent factors considered in the analysis, namely the block (the different weight classes) and the treatment (diets), and each level of the treatment occurred with each level of the block. However, there was only one factor which was truly of interest to our experimenter, the treatment (diets).

The techniques discussed for the randomized block design can be extended to the situation where there are two factors of interest. The main features of the factorial design are: