12.2 Exercises

Basic Concepts

1. **a.** What are the random variables in the test statistics in this section?

b. Why are they random variables?

c. Which sampling distribution do we use in the formulation of the test statistic when comparing two population means with unknown population variances or standard deviations?

d. What are the properties of the distributions referenced in part **c.**?

2. What assumptions are necessary to perform a hypothesis test for the difference between two independent population means when the population variances or standard deviations are unknown?

3. What is the test statistic for a hypothesis test about two population means when the population variances or standard deviations are unknown? How does this statistic differ from the test statistic used when the population standard deviation is known?

4. What is a pooled variance? Why is it used?

Exercises

5. Determine the critical value(s) of the test statistic for each of the following tests for the comparison of two population means. Assume the population standard deviations are unknown but equal, and $n_1 = n_2 = 40$.

a. Left-tailed test, $\alpha = 0.04$

b. Right-tailed test, $\alpha = 0.08$

c. Two-tailed test, $\alpha = 0.02$

6. A luxury car dealer is considering two possible locations for a new auto mall. The rent on the south side of town is cheaper. However, the dealer believes that the average household income is significantly higher on the north side of town. The dealer has decided that he will locate the new auto mall on the north side of town if the results of a study that he commissioned show that the average household income is significantly higher on the north side of town. Let Population 1 be the North Side of town and Population 2 be the South Side of town. The results of the study are as follows.

Income (Thousands of Dollars)			
	n	$\bar{x}$	s
North Side	35	72	10
South Side	40	68	5

a. Calculate a 90% confidence interval for the difference in average income between the north and south sides of town, assuming that the population standard deviations are equal. Interpret the interval.

b. Based on the study, will the auto dealer decide to locate the new auto mall on the north side of town? Use $\alpha = 0.05$.

7. An internal auditor for Tiger Enterprises has been asked to determine if there is a difference in the average amount charged for daily expenses by two top salespeople, Mrs. Ellis and Mr. Ford. The auditor randomly selects 45 days and determines the daily expenses for each of the salespeople. Let Population 1 be the daily expenses of Mrs. Ellis and Population 2 be the daily expenses of Mr. Ford.

Expenses (Dollars)			
	n	$\bar{x}$	s
Mrs. Ellis	45	\$55	\$8
Mr. Ford	45	\$60	\$3

a. Calculate a 95% confidence interval for the difference in the average amounts charged for daily expenses between Mrs. Ellis and Mr. Ford, assuming that the population standard deviations are not equal. Interpret the interval.

b. Based on the survey, can the auditor conclude that there is a difference in the average amounts charged for daily expenses by the two top salespeople? Use $\alpha = 0.05$.

c. Explain how the 95% confidence interval in part **a.** would lead you to make the same decision that was made in part **b.**

8. The military has two different programs for training aircraft personnel. A government regulatory agency has been commissioned to evaluate any differences that may exist between the two programs. The agency administers standardized tests to randomly selected groups of students from the two programs. The results of the tests for the students in each of the programs are as follows. Let Population 1 be the test results of students in training program A and Population 2 be the test results of students in training program B.

Military Training Programs			
	n	$\bar{x}$	s
Program A	50	85	10
Program B	55	87	9

a. Calculate a 99% confidence interval for the difference between the average scores of the two military programs, assuming that the population standard deviations are equal. Interpret the interval.

b. Can the agency conclude that there is a difference in the average test scores of students in the two programs? Use $\alpha = 0.01$.

9. Tom Sealack, an electrical engineer with the Navy, has been asked to determine if a new battery that has been offered to the Navy (at a reduced price) has a shorter average life than the battery they are currently using. He randomly selects batteries of each type and allows them to run continuously so that he can measure the time until failure for each battery. The results of the test are as follows. Assume that the population standard deviations are equal. Let Population 1 be the battery life of the new battery and Population 2 be the battery life of the old battery.

Battery Life (Hours)			
	n	$\bar{x}$	s
New Battery	35	700	30
Old Battery	35	710	35

a. Does the data suggest at $\alpha = 0.10$ that the time until failure for the new battery is significantly less than the time until failure for the old battery?

b. Calculate the *P*-value for the test in **a.**

c. Based on the *P*-value, would the decision change at $\alpha = 0.05$?

10. The City Bank believes that checking account balances are significantly larger for customers who are aged 40 to 49 than those who are aged 30 to 39. To investigate this belief, they randomly select customers from each age group and determine the average daily account balance for each customer for the current month. The results of the study are as follows. Assume that the population standard deviations are not equal. Let Population 1 be the 30 - 39 age group and Population 2 be the 40 - 49 age group.

Checking Account Balances (Dollars)			
Age Group	***n***	$\bar{x}$	***s***
30 – 39	200	\$2500	\$550
40 – 49	150	\$3500	\$950

a. Does the data suggest at $\alpha = 0.05$ that the average daily account balances are significantly higher for the 40 to 49 age group than the 30 to 39 age group?

b. Calculate the *P*-value for the test in **a.**

c. Based on the *P*-value, would the decision change at $\alpha = 0.10$?

11. Determine the critical value(s) of the test statistic for each of the following tests for the comparison of two population means where the assumptions of normality have been satisfied and the population standard deviations are unknown but equal.

a. Left-tailed test, $\alpha = 0.05$, $n_1 = 10$, $n_2 = 15$

b. Right-tailed test, $\alpha = 0.10$, $n_1 = 8$, $n_2 = 12$

c. Two-tailed test, $\alpha = 0.01$, $n_1 = 5$, $n_2 = 7$

12. Determine the critical value(s) of the test statistic for each of the following tests for the comparison of two population means where the assumptions of normality have been satisfied and the population standard deviations are unknown and unequal.

a. Left-tailed test, $\alpha = 0.025$, $n_1 = 13$, $n_2 = 25$

b. Right-tailed test, $\alpha = 0.005$, $n_1 = 7$, $n_2 = 18$

c. Two-tailed test, $\alpha = 0.10$, $n_1 = 15$, $n_2 = 15$

13. A cereal manufacturer has advertised that its product, Fiber Oat Flakes, has a lower fat content than its competitor, Bran Flakes Plus. Because of complaints from the manufacturers of Bran Flakes Plus, the FDA has decided to test the claim that Fiber Oat Flakes has a lower average fat content than Bran Flakes Plus. Several boxes of each cereal are selected and the fat content per serving is measured. The results of the study are as follows. Assume that the population variances are approximately equal and that the assumptions of normality have been satisfied. Let Population 1 be the fat content of Fiber Oat Flakes and Population 2 be the fat content of Bran Flakes Plus.

Fat Content (Grams)			
	n	$\bar{x}$	s
Fiber Oat Flakes	16	5	1
Bran Flakes Plus	15	6	2

a. Calculate a 90% confidence interval for the difference in average fat content between Fiber Oat Flakes and Bran Flakes Plus. Interpret the interval.

b. Does the study performed by the FDA substantiate the claim made by the manufacturer of Fiber Oat Flakes at $\alpha = 0.10$?

c. What assumptions must be made in order to calculate the confidence interval in part **a.** and perform the hypothesis test in part **b.**?

14. A large construction company would like to expand its operations into a new geographic area. The company has narrowed the choice of locations down to two cities. A major consideration in deciding between the two cities will be the average hourly wage they must pay for general laborers. The company randomly selects laborers from each city and determines their hourly wage with the following results. Assume that the population variances are approximately equal and that the assumptions of normality have been satisfied. Let Population 1 be City A hourly wages and Population 2 be City B hourly wages.

Hourly Wages (Dollars)			
	n	$\bar{x}$	s
City A	20	\$17	\$3
City B	20	\$14	\$2

a. Calculate a 99% confidence interval for the difference in average hourly wage between City A and City B. Interpret the interval.

b. Does the data indicate that there is a significant difference in hourly wages at $\alpha = 0.05$?

c. Calculate the P-value for the test performed in part **b.**

d. What assumptions must be made in order to calculate the confidence interval in part **a.** and perform the hypothesis test in part **b.**?

15. A Hollywood studio believes that a movie that is considered a drama will draw a larger crowd on average than a movie that is a comedy. To test this theory, the studio randomly selects several movies that are classified as dramas and several movies that are classified as comedies and determines the box office revenue for each movie. The results of the survey are as follows. Assume that the population variances are not equal and that the assumptions of normality have been satisfied. Let Population 1 be drama box office revenues and Population 2 be comedy box office revenues.

Box Office Revenues (Millions of Dollars)			
	n	$\bar{x}$	s
Drama	15	180	50
Comedy	13	150	30

a. Calculate a 95% confidence interval for the difference in average revenue at the box office for drama and comedy movies. Interpret the interval.

b. Does the data substantiate the studio's belief that dramas will draw a larger crowd on average than comedies at $\alpha = 0.01$?

c. Calculate the *P*-value for the test you conducted in part **b.**

d. What assumptions must be made in order to calculate the confidence interval in part **a.** and to perform the hypothesis test in part **b.**?

16. *Consumer Magazine* is reviewing the top of the line amplifiers produced by two major stereo manufacturers. One of the most important qualities of the amplifiers is the maximum power output. Brand A has redone their internal design and claims to have a higher maximum power level than Brand B. To test this claim, *Consumer Magazine* randomly selects amplifiers from each brand and determines the maximum power output. The results of the test are as follows. Assume that the population variances are approximately equal and that the assumptions of normality have been satisfied. Let Population 1 be Brand A amplifiers and Population 2 be Brand B amplifiers.

Amplifier Power Output (Watts)			
	n	$\bar{x}$	s
Brand A	12	800	25
Brand B	10	780	25

a. What assumptions must be made in order to perform the hypothesis test?

b. Does the data substantiate the claim that the Brand A amplifier has a higher average maximum power output than Brand B at $\alpha = 0.05$?

17. The State Environmental Board wants to compare pollution levels in two of its major cities. Sunshine City thrives on the tourist industry and Service City thrives on the service industry. The environmental board randomly selects several areas within the cities and measures the pollution levels in parts per million with the following results. Assume that the population variances are approximately equal and that the assumptions of normality have been satisfied. Let Population 1 be Sunshine City and Population 2 be Service City.

Pollution Levels (ppm)			
	n	$\bar{x}$	s
Sunshine City	15	8.5	0.57
Service City	10	7.9	0.50

a. What assumptions must be made in order to perform a hypothesis test for the difference between these two population means?

b. Will the State Environmental Board conclude at $\alpha = 0.01$ that Service City has a lower pollution level on average than Sunshine City?

c. Repeat part **b.**, assuming that the population variances are not equal.

d. Compare the results of part **b.** and part **c.**