12.1 Exercises

Basic Concepts

1. What questions are we interested in answering when comparing two population means?
2. What is an independent experimental design?
3. How does the determination of the critical value(s) for a two-sample hypothesis test differ from a one-sample hypothesis test?
4. What conditions are necessary to use the normal distribution to perform a hypothesis test for the difference between two independent population means?
5. Describe how the data guides us to a conclusion in testing a hypothesis about the difference in population means?

Exercises

6. A researcher compares the effectiveness of two different instructional methods for teaching anatomy. A sample of 134 students using Method 1 produces a testing average of 54.6. A sample of 150 students using Method 2 produces a testing average of 53.4. Assume the population standard deviation is known to be 5.1 for Method 1 and 12.47 for Method 2. Determine the 90% confidence interval for the true difference between testing averages for students using Method 1 and students using Method 2.

7. A student researcher compares the ages of cars owned by students and cars owned by faculty at a local state college. A sample of 109 cars owned by students had an average age of 7.81 years. A sample of 126 cars owned by faculty had an average age of 5.93 years. Assume the standard deviation is known to be 2.70 years for age of cars owned by students and 2.18 years for age of cars owned by faculty. Determine the 95% confidence interval for the difference between the true mean ages for cars owned by students and faculty. Let Population 1 be cars owned by students and Population 2 be cars owned by faculty.

8. Red Auerbach, a Hall of Fame coach of the Boston Celtics, is quoted as saying, "You can't teach height."[1] Although the heights of NBA point guards have been increasing over time, this trend may not apply to all positions. To investigate whether there is a difference in the heights of NBA players over time, data from the rosters of the NBA teams during the 1991 and 2021 seasons were compared. The sample mean height of 28 players who played during the 1991 season was 79.65 inches and a sample mean height of 26 players from the 2021 season was 78.81 inches. Height is commonly assumed to follow a normal distribution with a population standard deviation of 3 inches. Let Population 1 be the 1991 season NBA players and Population 2 be the 2021 season NBA players.
   - **a.** Construct a 90% confidence interval for the difference between the true mean heights of NBA players from the 1991 season and the 2021 season.
   - **b.** Does the interval contain the value of zero? Explain what this means in the context of NBA player heights.
   - **c.** What are possible sources of uncertainty about the claim about the difference in heights of NBA players over the past several decades?

9. *Popular Science* (Vol. 242, No. 3) reported the results of a comparison of several popular minivans.[2] One of the features that they compared was the time required to accelerate from 0 to 60 miles per hour in seconds. The Dodge Grand Caravan ES was able to accelerate from 0 to 60 mph in 11.3 seconds, on average. The Volkswagen Eurovan took 16.5 seconds on average to accelerate from 0 to 60 mph. Suppose that 35 minivans of each type were tested and that the population standard deviation of the times required to accelerate from 0 to 60 for each type of minivan is expected to be 4 seconds, based on historical data. Let Population 1 be the acceleration times of the Dodge Grand Caravan ES and Population 2 be the acceleration times of the Volkswagen Eurovan.

a. Calculate a 95% confidence interval for the difference in average acceleration time between the two types of minivans. Interpret the interval.

b. Does the data suggest that there is a significant difference in the time required to accelerate from 0 to 60 between the two types of minivans at $\alpha = 0.05$?

c. What assumptions did you make about the time required to accelerate from 0 to 60 mph in calculating the confidence interval in part **a.** and for performing the test in part **b.**?

10. Determine the critical value(s) of the test statistic for each of the following tests for the comparison of two population means. Assume the population standard deviations are known and $n_1 = n_2 = 40$.

a. Left-tailed test, $\alpha = 0.05$

b. Right-tailed test, $\alpha = 0.10$

c. Two-tailed test, $\alpha = 0.01$

11. A researcher compares two compounds (A and B) used in the manufacture of car tires that are designed to reduce braking distances. The mean braking distance at a speed of 25 mph for tires made with compound A is 61 feet, with a population standard deviation of 8.5. The mean braking distance for tires made with compound B is 66 feet, with a population standard deviation of 14.7. Suppose that a sample of 65 braking tests are performed for each compound. Using these results, test the claim that the braking distance for tires using compound A is shorter than the braking distance when compound B is used. Let μ_1 be the true mean braking distance corresponding to compound A and μ_2 be the true mean braking distance corresponding to compound B. Use the 0.1 level of significance.

12. A medical researcher wants to compare the pulse rates of smokers and non-smokers. He believes that the pulse rate for smokers and non-smokers is different and wants to test this claim at the 0.05 level of significance. A sample of 70 smokers has a mean pulse rate of 68, and a sample of 82 non-smokers has a mean pulse rate of 71. The population standard deviation of the pulse rates is known to be 8 for smokers and 9 for non-smokers. Let μ_1 be the true mean pulse rate for smokers and μ_2 be the true mean pulse rate for non-smokers.

13. A certain test preparation course is designed to improve students' SAT Math scores. The students who took the prep course have a mean SAT Math score of 512, while the students who did not take the prep course have a mean SAT Math score of 504. Assume that the population standard deviation of the SAT Math scores for students who took the prep course is 41.9 and for students who did not take the prep course is 33.4. The SAT Math scores are taken for a sample of 74

students who took the prep course and a sample of 92 students who did not take the prep course. Conduct a hypothesis test of the claim that the SAT Math scores for students who took the prep course is higher than the SAT Math scores for students who did not take the prep course. Let μ_1 be the true mean SAT Math score for students who took the prep course and μ_2 be the true mean SAT Math score for students who did not take the prep course. Use a 0.10 level of significance.

14. The manager of a city bus system is trying to assess commuter use of a particular bus line. He suspects that, on a weekday morning at 8 a.m., more passengers ride that line on the Northbound route than the Southbound route. The manager asks his Northbound driver and his Southbound driver to count how many passengers are on their 8 a.m. weekday routes for two weeks. The resulting data is shown below. Assume that the population standard deviation for passengers on the Northbound route is 2.6 and that the population standard deviation for passengers on the Southbound route is 3.5. Is there sufficient evidence at the 0.05 level of significance to say that, on a weekday morning at 8 a.m., more passengers ride the bus line on the Northbound route than the Southbound route? Assume that both populations are approximately normally distributed. Let passengers on the Northbound route be Population 1 and let passengers on the Southbound route be Population 2.

Northbound	38	32	35	34	31	36	33	37	36	31
Southbound	34	35	30	34	33	32	28	28	25	35

12.2 Inference about Two Population Means: Independent Samples, σ_1 and σ_2 Unknown

In empirical research it is unlikely that population means or variances of interest will be known. It is still possible to make comparisons between two population means if the population means and standard deviations are unknown when the populations are (approximately) normally distributed or the samples are large. In this section we will examine two methods of comparing population means when the variances of the population are unknown. The first method will assume the variances of the populations are unknown but equal, and the second method will assume the variances are unknown and not equal.

Inferences About the Mean of Two Independent Populations, σ_1 and σ_2 Unknown and Assumed Equal

Inference about two means with equal but unknown variances is a common statistical method used to test whether there is a significant difference between the means of two independent populations. This technique is used when we have two independent samples