In the previous two examples, we have considered a two-sided alternative and a one-sided "greater than" alternative. When considering a one-sided "less than" alternative, the procedure is very similar to that of a one-sided "greater than" alternative. The null hypothesis will be rejected if the calculated value of the test statistics, z, is less than the critical value, z_α, for the specified level of significance.

One-Sided "Less Than" Alternatives

For tests which are based on a test statistic which has a standard normal distribution and "less than" alternatives, find the value of z that *cuts off* α worth of probability in the left-hand tail of the distribution. The critical values for "less than" alternative hypotheses are given in Table 11.2.4 for typical values of α.

Table 11.2.4 – Critical Values of the *z*-Test Statistic for One-Sided (Less Than) Alternatives

Significance Level	Definition of Ordinary Variability	$-z_\alpha$
0.20	Upper 80% of the distribution	–0.84
0.10	Upper 90% of the distribution	–1.28
0.05	Upper 95% of the distribution	–1.645
0.01	Upper 99% of the distribution	–2.33

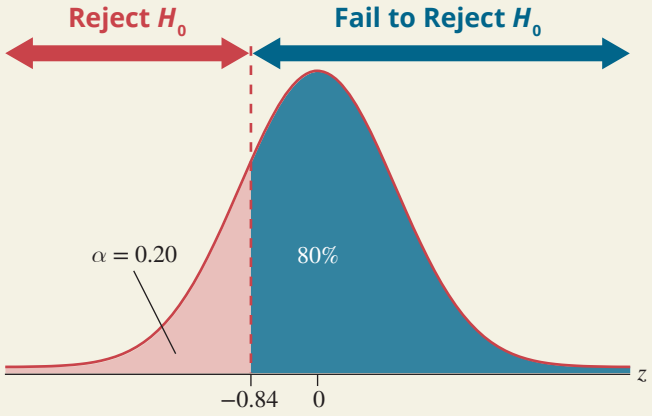

Figure 11.2.4

The figure above shows the rejection region for a test with a one-sided "less than" alternative hypothesis, and a significance level of 0.20. For this test, the null hypothesis will be rejected if the calculated value of the test statistic is less than –0.84.

The P-value is given by $P(z \leq z_0)$, where z_0 is the observed value of the test statistic.

PROCEDURE

11.2 Exercises

Basic Concepts

1. What is the rationale for the z-statistic?
2. Describe the distribution of the z-test statistic.

3. What are critical values? How do critical values influence the decision rule in the hypothesis testing procedure?
4. Suppose a null hypothesis was rejected at $\alpha = 0.05$. Would it be rejected at 0.10? Explain.
5. What is a *P*-value?

Exercises

6. Determine the critical value(s) of the test statistic for each of the following tests for the population mean with σ known.
   a. Left-tailed test, $\alpha = 0.01$
   b. Right-tailed test, $\alpha = 0.10$
   c. Two-tailed test, $\alpha = 0.05$

7. For each of the following combinations of the *P*-value and α, decide whether you would reject or fail to reject the null hypothesis.
   a. *P*-value = 0.0839, $\alpha = 0.05$
   b. *P*-value = 0.0174, $\alpha = 0.02$
   c. *P*-value = 0.0444, $\alpha = 0.10$
   d. *P*-value = 0.0374, $\alpha = 0.01$

8. Consider the following hypothesis tests for the population mean with σ known. Compute the *P*-value for each test and decide whether you would reject or fail to reject the null hypothesis at $\alpha = 0.05$.
   a. $H_0: \mu = 15,\ H_a: \mu > 15,\ z = 1.58$
   b. $H_0: \mu = 1.9,\ H_a: \mu < 1.9,\ z = -2.25$
   c. $H_0: \mu = 100,\ H_a: \mu \neq 100,\ z = 1.90$

9. Consider the following hypothesis tests for the population mean with σ known. Compute the *P*-value for each test and decide whether you would reject or fail to reject the null hypothesis at $\alpha = 0.01$.
   a. $H_0: \mu = 10,\ H_a: \mu > 10,\ z = 2.00$
   b. $H_0: \mu = 82,\ H_a: \mu < 82,\ z = -2.45$
   c. $H_0: \mu = 100, H_a: \mu \neq 100,\ z = 2.70$

10. The Tesla S electric vehicle (EV) can travel on average up to 405 miles on a single battery charge. Another manufacturer claims that its EV can travel further on a full battery charge. A random sample of 50 vehicles from the new manufacturer produces a sample mean of 408 miles. Test the hypothesis that the mean distance traveled by the manufacturer's EV is greater than 405 at $\alpha = 0.05$. Assume that the population standard deviation is 12 miles.

11. Hurricane Ian swept through southeastern Florida causing billions of dollars of damage. Because of the severity of the storm and the type of residential construction used in this semitropical area, there was some concern that the average claim size would be greater than the historical average hurricane claim.

Historically, the average claim size was \$24,000 with standard deviation \$2400.[3] Several insurance companies collaborated in a data gathering experiment. They randomly selected 84 homes and sent adjusters to settle the claims. In the sample of 84 homes, the average claim was \$26,000.

a. What is the population being studied?

b. What statistical measure should you use in your hypothesis?

c. State your hypotheses.

d. Test the hypothesis at the 0.01 level.

e. Is there overwhelming evidence (at the 0.01 level) that home damage is greater than the historical average? Write your conclusion in the context of the original problem.

12. For adults, a cholesterol value under 200 mg/dl is preferred. High cholesterol values over a period of time can result in a stroke or heart attack. A random sample of 10 cholesterol readings for a patient has an average of 204 mg/dl. The population standard deviation of the test is 6 mg/dl.

a. Is there overwhelming evidence to conclude that the patient's cholesterol level is above 200 mg/dl at a 0.05 significance level?

b. What is the lowest cholesterol average that would allow the patient to conclude that their cholesterol level is above 200 mg/dl?

13. A horticulturist working for a large plant nursery is conducting experiments on the growth rate of a new shrub. Based on previous research, the horticulturist feels the average daily growth rate of the new shrub is 0.2 cm per day with a standard deviation of 0.03 cm. A random sample of 45 shrubs has an average growth of 0.15 cm per day. Will a test of hypothesis at the 0.05 significance level support the claim that the growth rate is less than 0.2 cm per day?

14. Government regulations restrict the amount of pollutants that can be released to the atmosphere through industrial smokestacks. To demonstrate that their smokestacks are releasing pollutants below the mandated limit of 5 parts per billion pollutants, REM Industries collects a random sample of 300 readings. The mean pollutant level for the sample is 4.85 parts per billion. The population standard deviation is known to be 0.30 parts per billion.

a. Does the data support the claim that the average pollutants produced by REM Industries are below the mandated level at a 0.01 significance level?

b. What assumption did you make in performing the test in part **a.**?

15. In 2022, 3.96 billion people used social media, which is more than half the population of the world.[4] The average time spent on social media around the world is 147 minutes a day. Jennifer's daily average time spent on social media for the last ten days was 132 minutes.

a. Perform a hypothesis test to determine if Jennifer's time spent on social media is significantly less than the average. Use $\alpha = 0.01$. Assume the population standard deviation is 32.5 minutes.

b. What assumption did you make in performing the test in part **a.**?