### Note

Between **Step 2** and **3** of the hypothesis testing procedure, it is typical to collect sample data from the population and prepare it for analysis. However, for the problems presented in this textbook, the required data has already been provided, so this step is not necessary for our purposes.

In real-world scenarios, gathering sample data can be a challenging undertaking. In fact, according to data science literature, approximately 80% of the time required to perform a statistical analysis is devoted to collecting, extracting, preparing, cleaning, and organizing the sample data, with only 20% of the time dedicated to the actual statistical analysis.

### Note

Note at **Step 4** there are two options; you can find the critical value of the test statistic or the *P*-value of the test statistic. Both methods will always produce equivalent results; meaning, the decision regarding the hypothesis test will always be the same with both methods. We will often cover both methods in an example to illustrate this. Even though we may show a critical value and a *P*-value, only one of these is required to make the decision to reject or fail to reject the null hypothesis. You or your instructor may have a preference of one method over another.

### Steps in the Hypothesis Test

**Step 1:** Determine the population parameter to be used and develop the null and alternative hypotheses.

**Step 2:** Specify the significance level $\alpha$.

**see sidebar*

**Step 3:** Validate the assumptions of the hypothesis test, identify the appropriate test statistic, and compute its value.

**Step 4:** Determine the critical value(s) or *P*-value.

| Critical Value Method | *P*-Value Method |
|---|---|
| Find the critical value(s). (It may help to draw a graph displaying the critical value(s), the rejection region, and the test statistic.) | Find the *P*-value based on the value of the test statistic and the alternative hypothesis. (It may help to draw a graph displaying the test statistic and *P*-value.) |

**Step 5:** Choose between the null and alternative hypotheses.

| Critical Value Method | *P*-Value Method |
|---|---|
| ·Reject $H_0$ if the test statistic is in the rejection region.<br>·Fail to reject $H_0$ if the test statistic is not in the rejection region. | ·Reject $H_0$ if *P*-value is $< \alpha$.<br>·Fail to reject $H_0$ if *P*-value is $\geq \alpha$. |

**Step 6:** State the conclusion in terms of the original question.

PROCEDURE

# 11.1 Exercises

## Basic Concepts

1. What is a hypothesis?
2. What is the first step in the test of a hypothesis?
3. Describe the common elements present in all hypothesis tests.

4. Summarize the difference between the null and alternative hypotheses.
5. Define and give an example of a one-sided alternative. How does this differ from a two-sided alternative?
6. Is there a way to be absolutely certain your decision is correct when performing a hypothesis test? Explain.
7. What are the three important things you must be able to do in order to be successful at formulating hypothesis testing problems?
8. Describe a Type I error.
9. Describe a Type II error.
10. Explain how Type I and Type II errors influence the construction of a hypothesis.
11. Can both Type I and Type II errors be controlled in the hypothesis testing procedure? Explain.
12. What is the level of the test?
13. Why is a Type II error difficult to express numerically?
14. In the hypothesis testing process, in what way does the data guide our decision-making?
15. Explain the relationship between induction and the hypothesis testing process.

## Exercises

16. The town mayor believes that more than 47% of the town residents favor annexation of a new community. How should she formulate the hypotheses to test her claim?

17. A chocolate chip manufacturer would like to know if its bag filling machine works correctly at the 450 gram setting. Assume the population is normally distributed. How should the manufacturer formulate the hypotheses to test if the bags are being overfilled?

18. A hospital director believes that 29% of the lab reports contain errors and feels an audit is required. A sample of 300 reports found 99 errors. Is there sufficient evidence at the 0.02 level to refute the hospital director's claim? State the null and alternative hypotheses for this test.

19. An engineer has designed a valve that will regulate water pressure on an automobile engine. The valve was tested on 140 engines and the mean pressure was 7.7 lbs/square inch. Assume the variance is known to be 0.64. If the valve was designed to produce a mean pressure of 7.9 lbs/square inch, is there sufficient evidence at the 0.10 level that the valve performs below the specifications? State the null and alternative hypotheses.

20. Using traditional methods, it takes 10.9 hours to receive a basic flying license. A new license training method using Computer Aided Instruction (CAI) has been proposed. Set up the hypotheses to test the claim at the 0.05 level that the new technique performs differently than the traditional method. State the null and alternative hypotheses.

21. Our environment is very sensitive to the amount of ozone in the upper atmosphere. The level of ozone normally found is 7.6 parts/million (ppm). A researcher believes that the current ozone level is higher than the normal level. Set up the hypotheses to test the researcher's claim.

**22.** An automobile manufacturer claims that their van has a 36 miles per gallon (MPG) rating. An independent testing firm has been contracted to test the MPG for this van. After testing 53 vans they found a mean MPG of 32.8 with a standard deviation of 0.3 MPG. Is there sufficient evidence at the 0.05 level that the vans underperform the manufacturer's MPG rating? State the null and alternative hypotheses for this test.

**23.** A restaurant owner believes that tardiness has become a problem with her staff. In past years around 5% of her employees showed up late for their shift. She believes that the current rate is much higher. How should she formulate the hypotheses to test her belief?

**24.** A sample of 800 computer chips revealed that 79% of the chips do not fail in the first 1000 hours of their use. The company's promotional literature claimed that more than 76% do not fail in the first 1000 hours of their use. Is there sufficient evidence at the 0.02 level to support the company's claim? State the null and alternative hypotheses for this test.

**25.** A lumber company is making boards that are 2784 mm tall. If the boards are too long they must be trimmed, and if they are too short they cannot be used. A sample of 31 boards is made, and it is found that they have a mean of 2779.6 mm with a standard deviation of 10 mm. Is there evidence at the 0.10 significance level that the boards are too short and unusable? State the null and alternative hypotheses for this test.

**26.** For the following situations, develop the appropriate $H_0$ and $H_a$ and state what the consequences would be for Type I and Type II errors.

**a.** The Standard Tire Company has introduced a new tire in Europe that will be guaranteed to last at least 50,000 kilometers. Standard Tire has hired an independent agency to determine if there is overwhelming evidence that their tires will last through the warranty period.

**b.** A fisheries scientist claims that tilapia fed once a day with a specified formula will grow at least 200 grams in 110 days. Test whether there is overwhelming evidence to support this claim.

**c.** A horticulturist is experimenting with a new type of basil to grow and sell at the local farmer's market. Regular basil plants require at least 55 days to germinate at a temperature of 20° Celsius. This new type of basil is touted as being fast growing, so she expects that it will germinate in less time, resulting in larger plants.

**27.** For the following situations, develop the appropriate $H_0$ and $H_a$ and state what the consequences would be for Type I and Type II errors.

**a.** A company that manufactures one-half inch bolts selects a random sample of bolts to determine if the diameter of the bolts differs significantly from the required one-half inch.

**b.** A company that manufactures safety flares randomly selects 100 flares to determine if the flares last at least three hours on average.

**c.** A consumer group claims that a new electric vehicle (EV) model gets significantly fewer miles on a single battery charge than advertised by the manufacturer. To confirm this claim, researchers randomly drive several of the EV vehicles of this particular model and measure the distance traveled after a single full battery charge.