

Solution

Using the results from the initial sample,

$$n = \left(\frac{1.96 \cdot 4.3}{0.25} \right)^2 \approx 1136.50$$

$n \approx 1137$. (Always round up to assure the required confidence.)

Notice that while the sample data values are being collected they can be used to improve the estimate of the population standard deviation. For example, suppose the sample standard deviation after sampling the first 100 observations was 4.1. Using this estimate of s instead of 4.3 results in a sample size of 1034 compared to the original specification of 1137. The notion of modifying the sample size estimate as additional data is observed can be applied at regular intervals during the sampling process until the estimate of the standard deviation stabilizes.

10.3 Exercises

Basic Concepts

1. Why is the assumption that the population standard deviation is known when estimating the population mean not very realistic?
2. What effect does knowing the standard deviation of the population have on the construction of the confidence interval?
3. What is the Student's t -distribution?
4. What is the parameter of the t -distribution? How is it calculated?
5. What is the value of having a confidence interval with a small width?
6. Can a confidence interval be constructed with a width of your choice? Explain.
7. What is the margin of error? What is the connection between the expression for the margin of error and the equation to determine the sample size?
8. What is the difference between the method of determining the sample size when σ is known versus when σ is unknown?

Exercises

9. Find the t -value such that 0.025 of the area under the curve is to the right of the t -value. Assume the degrees of freedom equal 13.
10. Find the t -value such that 0.01 of the area under the curve is to the right of the t -value. Assume the degrees of freedom equal 21.
11. Find $t_{\alpha/2, df}$ for the following combinations of α and n .

<ol style="list-style-type: none"> a. $\alpha = 0.05, n = 15$ b. $\alpha = 0.01, n = 20$ c. $\alpha = 0.10, n = 8$ 	<ol style="list-style-type: none"> d. $\alpha = 0.05, n = 12$ e. $\alpha = 0.01, n = 18$ f. $\alpha = 0.10, n = 22$
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12. A random sample, consisting of the values listed below, was taken from a normally distributed population. Assuming the standard deviation of the population is unknown, construct a 99% confidence interval for the population mean.

27.4	26.5	25.7	31.4
28.2	21.9	16.3	22.7
18.8	34.4	29.2	20.5

13. Construct an 80% confidence interval for the mean of a normal population assuming that the values listed below comprise a random sample taken from the population. The population standard deviation is unknown.

83.9	87.4	65.2	86.0	73.1
80.3	92.7	87.5	69.3	77.5
91.9	71.1	79.1	72.4	88.2

14. An FDA representative randomly selects 8 packages of ground chuck from a grocery store and measures the fat content (as a percent) of each package. The resulting measurements are given below.

13%	12%	14%	17%
15%	16%	18%	15%

- a. Calculate the sample mean and the sample standard deviation of the fat contents.
 - b. Construct a 90% confidence interval for the true mean fat content of all the packages of ground beef.
 - c. What assumption did you make about the fat content in constructing your interval?
15. A hospital would like to determine the mean length of stay for its patients having abdominal surgery. A sample of 15 patients revealed a sample mean of 6.4 days and a sample standard deviation of 1.4 days.
- a. Find a 95% confidence interval for the mean length of stay for patients with abdominal surgery.
 - b. Interpret this interval and state any assumptions that were made in the construction of the interval.
16. An independent group of food service personnel conducted a survey on tipping practices in a large metropolitan area. They collected information on the percentage of the bill left as a tip for 25 randomly selected bills. The average tip was 18.3% of the bill with a standard deviation of 2.7%.
- a. Construct an interval to estimate the true average tip (as a percent of the bill) with 99% confidence.
 - b. Interpret the interval, and state any assumptions that were made in the construction of the interval.

17. A travel agent is interested in the average price of a hotel room during the summer in a resort community. The agent randomly selects 15 hotels from the community and determines the price of a regular room with a king size bed. The average price of the room for the sample was \$160 with a standard deviation of \$30.
- Construct an interval to estimate the true average price of a regular room with a king size bed in the resort community with 90% confidence.
 - Interpret the interval, and state any assumptions that were made in the construction of the interval.
18. A technician working for the Chase-National Food Additive Company would like to estimate the preserving ability of a new additive. This additive will be used for Auntie's brand preserves. Based on past tests, it is believed that the time to spoilage for this additive has a standard deviation of 6 days. To be 90% confident of the true mean time to spoilage, what sample size will be needed to estimate the mean time to spoilage with an accuracy of one day?
19. A computer software company would like to estimate how long it will take a beginner to become proficient at creating a graph using their new spreadsheet package. Past experience has indicated that the time required for a beginner to become proficient with a particular function of the new software product has an approximately normal distribution with a standard deviation of 15 minutes. Find the sample size necessary to estimate the true average time required for a beginner to become proficient at creating a graph with the new spreadsheet package to within 5 minutes with 95% confidence.
20. A food truck vendor is evaluating a downtown location by counting the number of people who walk past the prospective location on a particular day during lunch time (i.e. 11:00 AM to 2:00 PM). A preliminary study has indicated a standard deviation of about 30 people per lunch period. How many lunch periods will be needed to estimate the average number of people who walk past the prospective location during the lunch period to within 9 people with 90% confidence?

10.4 Estimating the Population Proportion

Recall that we discussed point estimation of proportions in Section 10.1. In this section we will develop a confidence interval for a population proportion based on the sampling distribution of the point estimate (see Section 9.3 for review).

Interval Estimation of a Population Attribute

The use of confidence intervals to apprise a decision maker of the reliability of estimates of a population mean can also be applied to estimating proportions. The random variable, \hat{p} , has a binomial distribution that can be approximated with a normal random variable.

Thus, the sample proportion, \hat{p} , is distributed normally with mean, p , and variance,

Note

Proportions have various equivalent formats, including fractions, decimals, and percentages.

$$\frac{3}{5} = 0.6 = 60\%$$