10.2 Exercises

Basic Concepts

1. **a.** What is an interval estimator?
   b. What is the difference between a point estimate and an interval estimate?
2. What is the distinction between probability and confidence?
3. What is the role of the z-value in the confidence interval expression?
4. Describe in words the ideas behind the construction of a confidence interval.
5. Suppose a 95% confidence interval for an estimate of a mean was 111 to 189. Explain what is wrong with the following expression: $P(111 < \mu < 189) = 0.95$.
6. What are the conditions required in order to construct a $100(1-\alpha)\%$ confidence interval using the expression $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$?
7. Describe the effect on the width of a confidence interval as each of the following increases: n, $1-\alpha$, α, $\bar{x}$.
8. What expression indicates the margin of error? Is this the same as the maximum error of estimation?
9. What is $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ an estimate of?

Exercises

10. Find $z_{\alpha/2}$ for the following levels of α.

a.	$\alpha = 0.05$	**d.**	$\alpha = 0.04$
b.	$\alpha = 0.01$	**e.**	$\alpha = 0.02$
c.	$\alpha = 0.10$	**f.**	$\alpha = 0.08$

11. Find $z_{\alpha/2}$ for the following confidence levels.

a.	98%	**d.**	96%
b.	94%	**e.**	88%
c.	92%	**f.**	85%

12. Construct a 90% confidence interval for the true mean of a normal population if a random sample of size 40 from the population yields a sample mean of 75 and the population has a standard deviation of 5.

13. A paint manufacturer is developing a new type of paint. Thirty panels were exposed to various corrosive conditions to measure the protective ability of the paint. The mean life for the samples was 168 hours before corrosive failure. The life of paint samples is assumed to be normally distributed with a population standard deviation of 30 hours. Find the 95% confidence interval for the mean life of the paint.

14. The chief purchaser for the State Education Commission is reviewing test data for a metal link chain which will be used on children's swing sets in elementary school playgrounds. The average tensile strength for a sample of 50 pieces of chain is

5000 psi. Based on past experience, the tensile strength of metal chains is known to be normally distributed with a standard deviation of 100 pounds. Estimate the actual mean tensile strength of the metal link chain with 99% confidence.

15. A research scholar wants to know how many times per week a strain of E. coli reproduces. From a sample of 476 organisms there was an average of 3 reproductions per hour. The population standard deviation is known to be 0.3 reproductions per hour. Construct a 95% confidence interval for the true population mean number of reproductions per hour for this bacteria.

16. An educational psychologist wishes to know the mean number of words that a third grader can read per minute. She wants to make an estimate at an 80% level of confidence. For a sample of 196 third graders, the mean words per minute was 27.1. Assume a population standard deviation of 3.2. Construct the confidence interval for the mean number of words that a third grader can read per minute and interpret the results.

10.3 Estimating the Population Mean, σ Unknown

In the last section we assumed that the population standard deviation is known. In practice this assumption is not very realistic, since the standard deviation describes variability about the mean. If the population standard deviation is known, the mean is usually also known, and there is no need to create an interval estimate for it. Why estimate something we already know? The methodology for creating the confidence interval when σ is unknown is quite similar to the methodology when σ is known but much more useful.

Interval Estimation of the Population Mean for a Normal Population with σ Unknown

If σ is not known and either the population is normally distributed or $n > 30$, the derivation of the confidence interval must be changed slightly.

Student's *t*-Distribution

Provided the population from which the sample is drawn is normally distributed or $n > 30$, the distribution of the quantity

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

where s is the standard deviation of the sample, has a **Student's *t*-distribution**.

FORMULA

The *t*-distribution is very much like the normal distribution (see Figure 10.3.1). It is a symmetrical, bell-shaped distribution with slightly thicker tails than a normal distribution. The shape of the *t*-distribution approaches the normal distribution as the **degrees of freedom**, the one parameter of the *t*-distribution, becomes larger.

William Sealy Gossett: The Student

Upon graduating from New College, Oxford with a strong understanding of mathematics, W.S. Gossett began working at the Guinness brewery in Dublin, Ireland. While working at Guinness, Gossett applied his statistical knowledge to find the best yielding varieties of barley, and in 1908, he developed the *t*-distribution. Few other statisticians at the time saw the merit in developing small-sample methods since most of their work required large data sets; however, Gossett was convinced of the importance of his work. Unfortunately, Guinness had prohibited its employees from publishing papers to protect trade secrets, and thus did not originally allow Gossett to publish his findings. After convincing the brewery that his statistical methods would be of no use to competing breweries, Guinness allowed Gossett to publish his conclusions on the *t*-distribution, but only under the pseudonym, "Student", to avoid issues with other staff members and company policies. To this day, Gossett's most noteworthy achievement is known simply as the "Student's *t*-distribution."